Oceanic torques on solid Earth and their effects on Earth rotation

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[1] We calculate that the oceanic torque vectors (x, y, z components) acting on the solid Earth using the Parallel Ocean Climate Model (POCM) output data at 3-day intervals for the period 1988–1995. This study examined three distinct types of torque, with different physical mechanisms: pressure, gravitational, and frictional. The pressure torque is further divided into two parts: that due to Earth’s ellipticity and that due to the ocean bottom topography. According to resultant time series the ellipticity pressure torque causes the largest effect for the x and y components. The gravitational torque and the ellipticity pressure torque are shown numerically to be exactly proportional by a factor of \(-0.49\). The topographic pressure torque is somewhat smaller in overall amplitude than the ellipticity-induced torque (the sum of the ellipticity pressure torque and gravitational torque). The ocean bottom frictional torque is found to be negligible in all three components. To validate the torque calculations, we compare the calculated oceanic torques with effective torque derived from the ocean angular momentum (OAM) variation, both from POCM. It is shown that the effective torque associated with the mass term of the OAM agrees remarkably well with the ellipticity-induced torque as required by theory. For the motion term the effective torque can be explained mostly by the topographic pressure torque in the intraseasonal periods, but the seasonal and longer-period components show significant differences between the two. The difference should theoretically be attributed to the exclusion of the wind frictional torque on the ocean. However, a comparison based on the NASA GEOS-1 atmospheric model shows that the wind frictional torque obtained is insufficient in power in the x and y components. Finally, in the application to the Earth rotation excitation it is found that both the ellipticity and topographic torques have comparable contributions to polar motion excitation for both seasonal and nonseasonal variations. For the length of day variation, our torque approach is presently inadequate because the oceanic contribution is relatively too small to be estimated in the presence of data and model errors.

INDEX TERMS: 1239 Geodesy and Gravity: Rotational variations; 1223 Geodesy and Gravity: Ocean/Earth/atmosphere interactions (3339); 4532 Oceanography: Physical: General circulation; 4504 Oceanography: Physical: Air/sea interactions (0312);
KEYWORDS: Earth rotation, oceanic torque, angular momentum

1. Introduction

[2] The Earth’s rotation varies slightly due to dynamic interactions between the solid Earth and global geophysical fluids [e.g., Chao et al., 2000]. The governing physical principle is the conservation of angular momentum, where the solid Earth continually exchanges a varying amount of its angular momentum with geophysical fluids such as the atmosphere and oceans. The solid Earth’s rotation, or simply the Earth rotation, varies as a result. This process is referred to as the excitation of Earth rotation variations, where the angular momentum variations in the geophysical fluids act as excitation functions.

[3] Earth rotation variations are described by a threedimensional (3-D) vector; these variations can be separated into the length of day (LOD) change (the axial component) and the polar motion (the two equatorial components). While it has been well known since the 1980s that the atmospheric angular momentum (AAM) explains a major portion of the Earth rotation excitations, especially for LOD, in the past decade, studies of the corresponding contributions of the oceanic angular momentum (OAM) began to appear. Lacking global data, such studies rely on ocean
general circulation models. Results were reported by Brosche et al. [1990, 1997], Frische and Sündermann [1990], Dickey et al. [1993], Ponte and Rosen [1994], Ponte [1997], Bryan [1997], among others. More recent studies revealed that ocean variabilities do play a significant role in exciting Earth rotation variations [Ponte et al., 1998; Furuya and Hamano, 1998; Marcus et al., 1998; Celaya et al., 1999]. Most recently, Johnson et al. [1999] examined the OAM variability using the Parallel Ocean Climate Model (POCM) [Semtner and Chervin, 1992] and demonstrated that the ocean accounts for a significant part of the non-AAM budget associated with both polar motion and LOD variations. Gross [2000] demonstrated the importance of OAM in maintaining the Chandler wobble of the polar motion.

The studies cited above are based on the so-called angular momentum approach, which has been prevalent in Earth rotation studies because of the ready accessibility of the data sets involved. Alternatively, one can study the geophysical fluids’ torques exerting on the solid Earth, which result in the exchange of angular momentum among the boundaries. The two approaches are dynamically equivalent [e.g., Munk and McDonald, 1960; Wahr, 1982]. However, there were limited studies only in the atmospheric case that used the torque approach to explain Earth rotation quantitatively [e.g., de Viron et al., 1999; B. V. Sanchez et al., Atmospheric torques on the solid Earth and oceans based on the GEOS-1 general circulation model, submitted to Journal of Geophysical Research, 2001, hereinafter referred to as Sanchez et al., submitted manuscript, 2001].

A torque is defined in rotational dynamics as the vector product of an acting force with its arm vector extending from a certain reference axis. Through torque, two bodies exchange angular momentum in such a way that the torque on each equals the rate of change of its angular momentum. In the absence of external torques the total angular momentum of a mechanical system is conserved.

In meteorology, classical studies of atmospheric torque date back to White [1949], followed by Newton [1971a, 1971b] and Oort and Bowman [1974]. Their main interest was to elucidate the atmospheric angular momentum (AAM) exchange through the “mountain torque” exerted by the pressure difference between both sides of mountains, the term being later extended to general topography on land.

In the case of the Earth, there is a variety of possible types of torque between Earth system components, torques differing in their forcing mechanisms, which are far from completely understood. A pioneering, comprehensive work by Wahr [1982] gave a theoretical basis for the torque approach and discussed the contribution of both atmospheric and oceanic torques to the Earth rotation.

Some recent papers have dealt with atmospheric torques acting on the solid Earth and oceans [Bell, 1994; Dehant et al., 1996; Madden et al., 1998; de Viron et al., 1999; Sanchez et al., submitted manuscript, 2001]. Dehant et al. [1996] summarized three different types of torque, pressure, gravitational, and frictional, while focusing on the pressure torque. de Viron et al. [1999] analytically formulated these atmospheric torques, calculated them using NASA GEOS-1 general circulation model and then successfully compared the equatorial components with the AAM derived from the same model. Sanchez et al. (submitted manuscript, 2001) made a similar calculation and comparison focusing on seasonal variations.

Less work has been devoted to the oceanic torques on the solid Earth. Peixoto and Oort [1992] described the physical picture of the “continental torque” exerted by the differences in ocean bottom pressures along continental margins and across submerged topography. Some recent studies have begun to explore the subject, most concentrating on the axial component of the rotation and on the mechanism by which the frictional torque generated by zonal winds on the ocean surface is transmitted to ocean floor [Ponte and Rosen, 1993, 1994; Segschneider and Sündermann, 1997; Bryan, 1997]. In particular, Segschneider and Sündermann [1997] computed the ocean’s contributions to interannual variations in length of day (axial torques) using the Hamburg large-scale ocean circulation model with 3.5° resolution and 11 vertical layers. Their results indicate that the frictional torque exerted by the wind on the ocean coincides with the torque exerted by the ocean on the solid Earth. Using the Geophysical Fluid Dynamics Laboratory Modular Ocean Model (MOM) with rigid lid, Bousinesq approximations, and variable resolution, Bryan [1997] showed that the balance between zonal wind stress and bottom pressure torques holds at all latitudes on seasonal timescales. These results indicate instantaneous transmission of the zonal wind frictional torque to the ocean bottom as bottom pressure torque at these timescales. More recently, Johnson [1998] presented preliminary results giving various oceanic torque components as calculated for global as well as hemispherical and regional ocean basins using POCM model output (see below). He pointed out the prospect that the closure of the torque budget can constrain estimates for the ocean bottom friction, which is far from known.

The purpose of the present paper is to examine general characteristics of the oceanic torque in all three dimensions using output from the Parallel Ocean Climate Model [Semtner and Chervin, 1992] and to validate the usefulness of the model calculation for the application of the torque approach to elucidating the oceanic effects on Earth rotation. We will first calculate the various components of the three types of oceanic torque (pressure, gravitational, and frictional). Then we examine the input-output balance of the torque with respect to the angular momentum obtained from the same model. Finally, on the basis of these examinations we will compare the excitation functions calculated from the oceanic torques with geodetic observations of Earth rotation variations in consideration of the AAM. As expected from the equivalence of the methodology and the validity of our torque calculations, the general features of the result are similar to those of Johnson et al. [1999], who uses the angular momentum approach with the same ocean model. Therefore in this paper, we only describe a summary of the result of our comparison using the torque approach and add some points on oceanic effects on the Earth rotation.

2. Ocean Model

We used the output of the Parallel Ocean Climate Model (POCM [Semtner and Chervin, 1992]). The POCM is a free-surface oceanic general circulation model (GCM) that performs dynamical calculations using the hydrostatic
and Boussinesq approximations. The model has 20 layers in the vertical profile with finer layering in the upper part of the ocean. The grid of the model is 0.4° in longitude and 0.4° cos(latitude) in latitude. The coastlines and bathymetry are prescribed at the local model resolution. The area covered extends from 75°S to 65°N and leaves out the Arctic Ocean.

[12] We used the specific version of POCM output designated as POCM_4B, which is forced by the daily surface wind and monthly heat flux fields produced by the European Centre for Medium-Range Weather Forecasts (ECMWF). A more detailed discussion of the model is given by Stammer et al. [1996]. The POCM_4B runs produced data every 3 days, beginning 3 January 1987. Prior to this date, the model had been “spun up” by a climatology wind field for 33 years. Stammer et al. [1996] made a comparison of the sea surface height output from this model with that derived from the TOPEX/POSEIDON altimetry, which showed a fairly good agreement.

[13] Johnson et al. [1999] used the POCM_4B output for the study of variability of OAM. This work noted certain crucial points in applying POCM to geodetic investigations: (1) a lack of water mass conservation; POCM, as with most current oceanic GCMs, conserves water volume rather than mass; (2) a significant underestimation of global oceanic variabilities; and (3) the fact that the (wind-driven) POCM does not include the atmospheric pressure variation as forcing. Another possible inadequacy is in the length of the initialization spin-up period and the transient “shock” of the model at the beginning when real wind field is injected into the model.

[14] In relation to point 1, Johnson [1998] specifically examined the mass variation of POCM and noted an apparent quadratic variation over several years in the total mass. He also found a similar trend in the OAM time series calculated from POCM and concluded that applying a simple quadratic polynomial fit removes the effect. In this paper, the same scheme will be applied to the time series when necessary.

[15] As Johnson et al. [1999] found, the original POCM_4B output was greatly reduced in grid density to facilitate data handling and computation. New grids with a horizontal spacing of 1° by 1°, and only six vertical layers were carefully reproduced by averaging the original parameters with little loss of physical significance of the original model. Grid boxes of the reduced model that do not contain at least 50% of the original ocean grid were flagged as land and excluded from the study. We will demonstrate later that the effect of this data reduction on our result is well tolerable. Only in the validation tests did we use the original fine grids, and the term “POCM” henceforth will refer to the reduced grid unless otherwise specified. The substantial physical parameter required in the present paper is the ocean bottom pressure which is calculated from the original model temperature and salinity output and then averaged into the reduced spacing as above.

3. Torque Calculations

[16] The derivation of the atmospheric/oceanic torques on the solid Earth from the equation of motion has been developed and summarized in several papers [Wahr, 1982; Barnes et al., 1983; Dehant et al., 1996; de Viron et al., 1999; Sanchez et al., submitted manuscript, 2001]. Here we basically follow the formulation given by Sanchez et al. (submitted manuscript, 2001) for the calculation of oceanic torques for the three different physical mechanisms: pressure, gravitation, and friction.

[17] Time series of the oceanic torques exerted on the solid Earth are then calculated using POCM output spanning the period 1988–1995. Note that each of the torques thus calculated has a large static part. We will simply remove the static part by subtracting the mean value from each time series. Physically these mean values are largely due to the existence of the nonoceanic parts of the Earth’s surface, namely, the continents, whose effect should balance the nonzero means of the oceanic torques.

3.1. Pressure Torque

[18] A pressure torque acting on the solid Earth from the ocean, L_p, is calculated by the following surface integral over the oceanic area:

\[ L_p = \int \mathbf{r} \times (-P_b \cdot \mathbf{n}) dS, \]

where \( \mathbf{r} \) is the radius vector from Earth’s center, \( P_b \) is the ocean bottom pressure field, and \( \mathbf{n} \) is the unit outward vector normal to the topographic surface. The integral can be expressed explicitly in terms of the ocean bottom topography function \( T \):

\[ \mathbf{r} \times (-P_b \cdot \mathbf{n}) = -P_b \left[ (\partial T/\partial \lambda) \cot \theta \cos \lambda + (\partial T/\partial \phi) \sin \lambda \right] \mathbf{i} + \left[ (\partial T/\partial \phi) \cot \theta \sin \lambda - (\partial T/\partial \phi) \cos \lambda \right] \mathbf{j} - (\partial T/\partial \lambda) \mathbf{k}, \]

where \( \theta \) is the colatitude and \( \lambda \) is the longitude. The three vector components are defined in the right-handed geographic coordinates \((x, y, z)\), which correspond to the axes toward the Greenwich Meridian, 90°E longitude, and the mean rotation axis, respectively.

[19] Note that the ocean bottom pressure adopted from POCM does not include any overlying atmospheric pressure effect (as stated above). We will consider that effect in a later discussion when incorporating AAM under the inverted-barometer assumption.

[20] As is customary, the Earth’s topography can be considered as the superposition of the dominant ellipticity plus shorter-wavelength “anomalies” relative to the reference spheroid. Thus we compute the two contributions separately in the following.

3.1.1. Ellipticity pressure torque

[21] The major part of the topography gradient comes from the ellipticity. It affects the two equatorial components \((x \text{ and } y)\) of the angular momentum and hence the polar motion:

\[ \partial T/\partial \phi = a [f \sin \theta - (3/4)f^2 \sin 4\phi], \]

where \( f = 1/298.275 \) is the Earth flattening parameter and \( a \) is the mean equatorial radius of the Earth. The \( z \) component vanishes because of the axial symmetry so that \((\partial T/\partial \lambda) = 0\).
Thus the ellipticity pressure torque has zero effect on the axial angular momentum or, equivalently, on length of day.

[22] Figure 1 shows the $x$ and $y$ components of the ellipticity pressure torque. The peak-to-peak amplitude amounts to $\sim 300$ Hadley units (HU, where $1 \, \text{HU} = 10^{18} \, \text{kg m}^2 \, \text{s}^{-2}$) in the $x$ component and somewhat less in the $y$ component. As will be seen from subsequent plots, this term gives the largest contribution to the total torque.

[23] The annual variation can be clearly seen, especially in the $x$ component. Both components show a slight drift with time. This, along with similar drifts in the other torques appearing later, seems to be an artifact arising from POCM’s lack of water mass conservation mentioned above. The $y$ component contains some anomalous signals during 1988 and 1995; they will be discussed later in connection with La Niña events.

[24] We now take a closer look at the spectral content of the torque. For this purpose, we applied the multitaper spectral method [Thomson, 1982]. This method yields robust spectral estimates where the spectral leakage is reduced in trading off with spectral resolution in an optimal fashion. Seven orthogonal tapers with a time-bandwidth product of $4\pi$ were adopted.

[25] Figure 2 shows the power spectra of the ellipticity pressure torque. To remove a drift and/or long-period fluctuations before applying the spectral analysis, all time series were detrended by removing a least squares fit of a quadratic polynomial. The following figures show the subseasonal spectral band between 10 and 100 days. The seasonal periods are not shown; prominent peaks at annual and semiannual periods would dwarf the subseasonal band of interest here. In the $x$ component, there is overall power around several tens of days, including broad spectral humps around 40–50 days and 60–70 days. In contrast, the $y$ component has much less power and only a small hump around 70–80 days.

3.1.2. Topographic pressure torque

[26] The remaining topographic contribution comes from the gradient of the ocean bottom topography relative to the reference spheroid. In view of the formulation, this torque corresponds to the so-called “mountain torque,” a term usually used in the atmospheric torque discussion [e.g., Wahr, 1982]. We should also mention here that the $z$ component of the topographic torque is supposed to correspond to the “continental torque” usually referred to as the axial oceanic torque [e.g., Peixoto and Oort, 1992].

[27] We used the ocean bottom topography given in the original POCM_4B model (horizontally averaged to $1^\circ$ by $1^\circ$ grid as indicated above). In contrast, de Viron et al. [1999] treated the atmospheric pressure torque in a single formulation in terms of the spherical harmonic expansion of the surface topography. Here we choose not to use the spherical harmonic expansion because such a representation would require much higher degrees and orders than for the atmosphere because of the ocean-continent geography.

[28] Figure 3 shows the topographic torque time series. There are clear seasonal signals in all three components. The $z$ component has a much smaller amplitude than the $x$ and $y$ components but a much larger drift. Peak-to-peak amplitudes amount to $\sim 80$ HU for the $x$ component and somewhat smaller for the $y$ component, which are about one third of the variability of the ellipticity pressure torque in Figure 1. The standard deviations of the topographic torque

![Figure 1](image1.png)  
*Figure 1.* The oceanic ellipticity pressure torque in the $x$ and $y$ component, 1988–1995, computed based on POCM_4B model output.

![Figure 2](image2.png)  
*Figure 2.* Power spectra of the $x$ and $y$ components of the ellipticity pressure torque for the subseasonal period range of 10–100 days.

![Figure 3](image3.png)  
*Figure 3.* Same as Figure 1 but for the topographic pressure torque and for all three components.
variations after removing a quadratic trend are listed in Table 1.

Figure 4 shows the multitaper power spectra from the topographic torque as in the case of ellipticity pressure torque. Several peaks are more prominent compared with the ellipticity pressure torque, although the overall power is much lower. It is noted that the $x$ and $y$ components show quite different features; in particular, the $x$ component has a clear peak around 40–50 days, which the $y$ component lacks. The $z$ component is not shown because there is little power other than at the seasonal periods.

Note that the topographic torque is calculated from the spatial gradient of the bottom topography, accentuating short-wavelength effects compared to the ellipticity effect. Here we examine the sensitivity of our result with respect to model grid density. We do so by comparing the topographic torque obtained above using our $1/176$ by $1/176$ reduced grid with one calculated from the original finer model grid for a 1990–1992 test period. Figure 5 shows a comparison between the time series of the averaged and original model for the $x$ component together with the difference of the two. As clearly seen, the first two curves agree with each other quite well, although the difference with an amplitude less than the 10-HU level still remains visible in the lower frequency. The difference for the $y$ component shows almost the same feature as in the lowest curve in Figure 5 both in amplitude and frequency. However, we should note here that for the $z$ component the difference is relatively more serious than those for the $x$ and $y$ components because its amplitude is significantly smaller as seen in Figure 3. We conclude here that the averaged model with reduced grids can well represent the original POCM_4B model for the topographic torque calculation but note that good care must be taken if a feature with an amplitude less than the 10-HU level is very crucial. We conducted further comparisons averaging up to $2/176$ by $2/176$ and obtained essentially the same result.

### 3.2. Gravitational Torque

Gravitational torque originates from the body force due to the nonradial part of the Earth’s gravitational field acting on the ocean. In the formulation of the gravitational torque for the atmospheric case it was shown that the volume integral over the whole atmospheric body can be reduced to the surface integral of the pressure field over the Earth and ocean surface [e.g., de Viron et al., 1999].

![Figure 4.](image1) Figure 4. Same as Figure 2 but for the topographic torque. The $z$ component is not shown because the power is very small other than the annual signal (compare Figure 3).

![Figure 5.](image2) Figure 5. Comparison of the topographic torque in the $x$ component calculated from different grid density for 1990–1992. The averaged grid is $1/176$ by $1/176$ spacing (used in this study), and the original grid from POCM_4B is $0.4^\circ$ (longitudinal) and $0.4^\circ \cos (\text{latitude})$ (latitudinal). The two series differ little. The difference of the two curves is shown in the bottom panel.

<table>
<thead>
<tr>
<th>Torque</th>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipticity-induced$^b$</td>
<td>67.3 (69.4)</td>
<td>44.1 (41.6)</td>
<td></td>
</tr>
<tr>
<td>Topographic$^a$</td>
<td>29.1 (27.7)</td>
<td>29.7 (28.8)</td>
<td>11.8 (11.1)</td>
</tr>
<tr>
<td>Bottom friction</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>OAM(mass)-derived</td>
<td>67.4</td>
<td>39.3</td>
<td></td>
</tr>
<tr>
<td>OAM(motion)-derived</td>
<td>27.1</td>
<td>28.2</td>
<td>3.8</td>
</tr>
<tr>
<td>OAM(motion)-derived minus topographic</td>
<td>15.4</td>
<td>14.0</td>
<td>-</td>
</tr>
<tr>
<td>Wind friction (GEOS-1)</td>
<td>5.8</td>
<td>6.1</td>
<td>11.5</td>
</tr>
</tbody>
</table>

$^a$Variations in Hadley units ($10^{19}$ newton meters).

$^b$For 8 years (1988–1995) with those for 3 years (1990–1992) in parentheses; otherwise, for 3 years.

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**Table 1.** Standard Deviation of Torque Variations After Removing a Quadratic Trend
parallel expression for the oceanic gravitational torque acting on the solid Earth, \( \mathbf{L}_{g} \), is

\[
\mathbf{L}_{g} = \int \mathbf{r} \times (-P'_{g} \mathbf{n}_{g})dS, \tag{4}
\]

where \( \mathbf{n}_{g} \) is the unit inward vector normal to the gravitational equipotential surface, but note that \( P'_{g} \) is the ocean bottom pressure field “minus” the effect from the atmospheric pressure field. In the calculation using the POCM, however, the atmospheric effect in \( P'_{g} \) is zero because of the absence of pressure forcing, and \( P'_{g} \) can be reduced to \( P_{g} \) as in section 3.1.

\[\text{[32]}\] In the calculation of the gravitational torque we used the normal vector of the gravitational potential field of EGM96 [Lemoine et al., 1998]. Although we included some higher degrees and orders in deriving the normal \( \mathbf{n}_{g} \), the only significant contribution comes from the degree 2 zonal component \((C_{20})\), or the Earth’s ellipticity; the higher-order terms are at least 3 orders of magnitude smaller.

\[\text{[33]}\] Figure 6 shows the \( x \) and \( y \) components of the gravitational torque; the \( z \) component is not shown because it has almost no power, which is evident from the ellipticity contribution pointed out above. A comparison with Figure 1 clearly shows that the two sets of curves have the same time pattern but a flip in sign. In fact, a closer examination shows that they are almost perfectly proportional to each other and the ratio of the gravitational torque to the ellipticity pressure torque is –0.487.

\[\text{[34]}\] The proportionality is also evident from their respective formulas. The geopotential normal does not include the centrifugal force contribution, which is present in the topographic normal. This is the root of their amplitude difference. de Viron et al. [1999] formed a reduced ratio by including only the \((2,0)\) terms for the geopotential field and the Earth’s figure. Our result simply exhibits this numerically. Therefore the total effect of the ellipticity, or “ellipticity-induced” torque, is the sum of the ellipticity pressure torque and the gravitational torque, which is equal to 51.3% of the ellipticity pressure torque in Figure 1. The standard deviations of the sum of the ellipticity-induced torque after removing a quadratic trend are listed in Table 1. The power spectra of the gravitational torque and/or ellipticity-induced torque are naturally the same as those of the ellipticity pressure torque in Figure 2 except for the absolute power.

### 3.3. Frictional Torque

\[\text{[35]}\] The frictional torque acting on the solid Earth, \( \mathbf{L}_{f} \), can be calculated by

\[
\mathbf{L}_{f} = \int (\mathbf{r} \times \mathbf{\tau}_{b})dS, \tag{5}
\]

where \( \mathbf{\tau}_{b} \) is the bottom friction vector from ocean to solid Earth. The integrand can be further expressed in terms of local stress components as follows:

\[
\mathbf{r} \times \mathbf{\tau}_{b} = r\left[-\tau_{0}\sin\lambda - \tau_{\lambda}\cos\lambda \cos\theta \mathbf{i} + (\tau_{0}\cos\lambda - \tau_{\lambda}\cos\theta \sin\lambda) \mathbf{j} + \tau_{\lambda}\sin\theta \mathbf{k}\right], \tag{6}
\]

where \( \tau_{0} \) and \( \tau_{\lambda} \) are components of \( \mathbf{\tau}_{b} \) in the colatitudinal and longitudinal directions, respectively. In the modeling of the POCM it is assumed that the bottom friction follows a quadratic drag law (R.Tokmakian, private communication, 2000):

\[
\tau_{b} = C_d \rho |\mathbf{u}| \mathbf{u}. \tag{7}
\]

\( C_d \) is the drag coefficient assuming the value of 0.001225, \( \rho \) is the water density, and \( \mathbf{u} \) is the horizontal velocity vector in the bottom layer.

\[\text{[36]}\] Figure 7 shows the resultant time series, and Table 1 gives their standard deviations but only for the period of 1990–1992 in this case. We see that the frictional torque is 2 orders of magnitude smaller than the ellipticity-induced and topographic torques and hence is negligible in later discussions. In fact, for the calculation of the ocean bottom friction, the applied \( C_d \) is generally very uncertain; the real situation is very complicated depending on the roughness of the ocean floor. Also, the bottom layer velocity estimation is unreliable. In addition, the bottom friction should include in
principle lateral viscosity terms that are not included in the POCM model calculation. This topic obviously awaits more investigations.

[37] In addition to the bottom friction in order to close the total balance for the oceanic torque, we ought to consider the wind frictional torque on the ocean surface, as it also acts on the ocean and exchanges angular momentum between the atmosphere and the ocean. This will be discussed in conjunction with OAM in section 4.

4. Balance With Ocean Angular Momentum OAM

[38] The torque \( \mathbf{L}_o \) exerted on the ocean by other Earth components change the OAM \( \mathbf{H} \). The relation can be expressed as

\[
\mathbf{L}_o = \frac{d\mathbf{H}}{dt} + \Omega \times \mathbf{H},
\]

where \( \Omega \) is the mean angular velocity of the Earth rotation [e.g., Munk and McDonald, 1960]. For the Earth, \( \mathbf{L}_o \) physically consists of the three oceanic torques (pressure, gravitational, and frictional torques) and the same set of torques from the atmosphere (and the same plus the additional electromagnetic torque from the core, which are of longer timescale and disregarded here).

[39] In this section we examine the balance of the oceanic torque with the associated OAM variation according to POCM. Note that the torques from the solid Earth included in \( \mathbf{L}_o \) are simply the negative of those discussed in section 3 under the definition of the forcing direction. Among those from the atmosphere, the pressure torque should not be considered here because the POCM contains no forcing by the atmospheric pressure. The gravitational torque caused by the atmospheric attraction is negligible because the atmospheric mass is much smaller than the solid Earth mass. The frictional torque from the atmosphere, i.e., wind frictional torque, will be discussed later in this section.

[40] We first calculate the OAM \( \mathbf{H} \) from POCM and derive the “effective” torque \( \mathbf{L}_o \) using equation (8). Then we compare this effective torque with the torques we computed above. The total OAM is defined as follows [e.g., Munk and McDonald, 1960]:

\[
\mathbf{H} = \mathbf{I} \cdot \Omega + \mathbf{h}.
\]  

The first term, in which \( \mathbf{I} \) represents ocean’s inertia tensor, is associated with the mass redistribution of the ocean water and is usually called the mass term or inertial term. The second term \( \mathbf{h} \) is the relative angular momentum associated with the ocean current velocity, often called the motion term or relative term.

[41] Wahr [1982] formulated the atmospheric and oceanic torques. He showed that the Earth’s ellipticity-induced torque (i.e., the ellipticity pressure torque plus the gravitational torque) is quantitatively equal to the effective torque from the mass term of the equatorial \((x, y)\) angular momentum under the low-frequency approximation. He argues that consequently the rest of the torques should therefore be reduced to the motion term.

[42] The low-frequency approximation made above is essentially equivalent to the negligence of the first (time derivative) term of equation (8) in the present paper. Our calculation of \( \mathbf{L}_o \) through equation (8) confirms that in our periods of interest, the first terms are by far smaller than the second term for both mass and motion terms and thus almost negligible. So on the basis of the Wahr’s [1982] assertion, we will make separate comparisons with respect to the mass and motion terms for the equatorial \((x, y)\) components, and refer to them as OAM(mass) and OAM(-motion), respectively. For the axial \((z)\) component we will make another comparison in section 4.3.

4.1. Mass Terms

[43] From the discussion above, one expects the effective torque derived from OAM(mass) to be equal to the ellipticity-induced torque. Figure 8 compares these two kinds of torques for both \( x \) and \( y \) components for the time period
Table 2. Qualitative Summary of Resultsa

<table>
<thead>
<tr>
<th>Torque</th>
<th>Torque Components</th>
<th>OAM-Derived Torque</th>
<th>OAM-Derived Torque Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equatorial (X, Y)</td>
<td>Axial (Z)</td>
<td></td>
</tr>
<tr>
<td>Friction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface ($L_{fs}$)</td>
<td>small</td>
<td>significant ($\approx -L_{pt}$)</td>
<td></td>
</tr>
<tr>
<td>Bottom ($L_{fb}$)</td>
<td>negligible</td>
<td>negligible</td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Topographic ($L_{pt}$)</td>
<td>significant</td>
<td>significant ($\approx -L_{fs}$)</td>
<td></td>
</tr>
<tr>
<td>Ellipticity ($L_{pe}$)</td>
<td>largest</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Gravitational</td>
<td>$-0.487 , L_{pe} \sim 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that all the torques are defined as those onto the ocean from the solid Earth and atmosphere (e.g., $L_{pe}$ is $-L_{pe}$ of section 3).

1990–1992. The two torques agree with each other remarkably well with insignificant differences shown in Figure 8 (bottom). The standard deviations of the OAM(mass)-derived torque variation are also given in Table 1. A comparison with the ellipticity-induced torque for the same period shown in parentheses in Table 1 also indicates the insignificant difference as shown in Figure 8.

It is interesting to note that the calculation of the OAM(mass)-derived torque is made from the mass distribution without any ellipticity expression, whereas that of the ellipticity-induced torque is entirely based on an explicit, analytical expression of the ellipticity and the gravity coefficient. The agreement shown here confirms the physical consistency of the POCM model calculation in terms of the spheroidal equilibrium condition. The result is consistent with those suggested by earlier work [Bell, 1994].

4.2. Motion Terms

Figure 9 shows the torque derived from OAM(motion) for the equatorial components (x, y) for 1990–1992. This torque should be compared with the sum of the topographic (pressure) torque and the frictional torque (Table 2). In section 4.1 we concluded that the ocean bottom frictional torque is rather small and hence negligible. Therefore we need to only consider the topographic torque that acts on the solid Earth (see Figure 3) and the wind frictional torque that only pertains to the atmosphere-ocean interaction.

Figure 9 also plots (as dashed curves) the topographic torque duplicated from Figure 3 but with the sign reversed. Two things to note here in the comparison: (1) a high correspondence in the non-seasonal (especially short period) signals and (2) amplitudes of the residuals significantly smaller than those for OAM(motion)-derived. These indicate that the topographic torque explains a significant portion of the equatorial OAM(motion)-derived torque with some substantial remnants mostly in the seasonal periods.

The difference between the two curves in Figure 9, or "residual," series are shown in Figure 10 as the solid curves. As clearly seen, the seasonal periods, especially the annual, dominate in the residual signals, especially in the x component. Their standard deviations along with those of the OAM(motion) ones are listed in Table 1. Theoretically, the residual signals represent the contribution from the wind frictional torque acting on the ocean surface, as discussed above. For the comparison with the wind frictional torque, one should naturally use the ECMWF wind field, which drives the POCM. However, since the ECMWF stress field is not immediately available, we adopt the wind frictional torque.
torque calculated by Sanchez et al. (submitted manuscript, 2001) from NASA's Goddard Earth Observing System version 1 (GEOS-1) atmospheric GCM [Schubert et al., 1993]. The dashed curves of Figure 10 give the GEOS-1 computed wind frictional torque; they are resampled at 3-day intervals (by averaging) from the original 3-hour values to match that of the POCM. It is seen that in the x and y components their amplitudes are much smaller than those of the residual (see Table 1). At this time it is not clear how much of this equatorial discrepancy is attributable to the difference in wind stress field between GEOS-1 and ECMWF.

4.3. Axial Component

[48] The axial (z) component of the effective torque derived from the sum of OAM(mass) and OAM(motion), referred to as OAM(sum), is shown in Figure 11 (top), with the same vertical scale as in Figure 9; its standard deviation is listed in Table 1. They show that the axial OAM(sum)-derived torque has much smaller amplitude than its equatorial counterparts. This corresponds to the fact that the contribution from the second term in equation (8) by far dominates the first term in the x and y components but the z component lacks the second term to a first-order approximation.

[49] We now discuss the OAM budget for the z component. Among the torques that we have already calculated, it is evident by geographical symmetry that the z component of the ellipticity-induced torque is identically zero. Also, we can neglect the ocean bottom frictional torque as in the previous discussion. On the other hand, the z component of the topographic torque has a significant amplitude, though substantially smaller than its x and y counterparts, compared with the OAM(sum)-derived torque as shown in Table 1 and Figures 3 and 11. Therefore the residual between the OAM(sum)-derived and topographic torques should be explained by the wind frictional torque on the ocean surface in order to close the budget.

[50] Figure 11 (bottom) shows the comparison between the residual (OAM(sum)-derived minus topographic torque) and the wind frictional torque from the GEOS-1 model, as was done in Figure 10 for the equatorial components but using the OAM(motion)-derived only. It demonstrates a good agreement between the two curves especially in the annual amplitude and phase, compared with those shown in Figure 10. However, the remaining discrepancy in Figure 11 is still significant especially at high frequencies. The reason cannot be identified at present but suggests plausible model differences in the wind stress fields as mentioned above for the equatorial case.

[51] Thus, for the axial z component the OAM-derived torque is very small, while there are significant topographic and wind frictional torques, which substantially cancel each other. This implies physically that the zonal wind frictional torque exerted on the ocean surface is transmitted to the solid Earth as a bottom topographic torque, with little contribution from the ocean itself (see Table 2). This is consistent with some previous results [e.g., Segschneider and Sündermann, 1997; Bryan, 1997].

[52] A comparison between the axial and equatorial components of the effective torques thus indicates that in effect, the ocean acts as a significant reservoir for the equatorial components, whereas it is merely a passage for the axial component of the angular momentum. This demonstrates the reason why the observed LOD variation can generally be explained mostly by atmosphere, while the polar motion cannot be.

5. Application to the Earth Rotation Study

[53] Ultimately, we want to examine the torque approach in studying Earth rotation dynamics. Thus the next step should be a comparison of the excitation functions calculated from the torques with the observed polar motion and/or LOD excitations. Johnson et al. [1999] have conducted such comparison using the POCM based on the angular momentum approach. We expect our comparison results would be similar to their’s, since we have examined the torque balance with OAM in section 4, which we showed to be fairly good except for the seasonal discrepancy presumably due to the differences in the wind frictional torque modeling. Here we report our results with respect to the equatorial components, i.e., the polar motion excitation, focusing on those specific to the torque approach. In
addition, we also describe oceanic effects on the polar motion that are not elaborated by Johnson et al. [1999]. For the axial component, i.e., the LOD variation, we found that our torque approach is inadequate, and hence inconclusive at present, because the oceanic contribution is too small to be estimated in the presence of the errors in the OAM budget discussion in section 4.3.

5.1. Methodology

The theoretical basis of our calculation of the excitation function of the polar motion through the torque approach is given by Wahr [1982]:

\[ \Omega^2 (C_m - A_m) \phi = i1.44[L(\text{ellip}) + L(\text{topo}) + L(\text{fric})] + (\text{pressure loading effect}), \] (10)

where \( C_m \) and \( A_m \) are the axial and equatorial moments, respectively, of inertia of the mantle and the complex-valued quantity \( \phi = \phi_a + i\phi_o \) is the polar motion excitation function, and similarly \( L = L_x + iL_y \) is the complex equatorial component of the torque. The last term of the right-hand side of equation (10) corresponds to the contribution of the pressure loading effect, which should equal \(-0.44L(\text{ellip})\) (canceling some of the effect in the first term) under the same low-frequency approximation made in section 4. We now compare the calculated \( \phi \) with that observed.

The observed polar motion used in our comparison is the SPACE98 series [Gross et al., 1998], from which we derive the “observed” excitation function \( \phi \) using the deconvolution equation (4a) of Wilson [1985] and adopting the values of 433.7 days for the Chandler resonance period and 49 days for the Chandler Q following Furuya and Chao [1996].

Just as Johnson et al. [1999] did, we consider the atmospheric contribution when investigating the oceanic excitation of the polar motion. We do so by adopting the AAM series calculated from the National Center for Environmental Prediction (NCEP) reanalysis data [Salstein et al., 1993; Kalnay et al., 1996]. We average the values over all 6-hour epochs in 3-day intervals, rather than picking an instantaneous value at a certain time of day. This scheme is reasonable since the oceanic response is usually slower than the atmospheric variation and acts as a low-pass filter. The AAM consists of two terms: the pressure, or “mass” term, and the wind, or “motion” term [e.g., Barnes et al., 1983]. The NCEP wind term includes wind contribution up to 10-mbar level. For the pressure term we use the series calculated under the inverted-barometer (IB) assumption that the ocean surface height responds isostatically to overlying atmospheric pressure variations. Since the POCM has no pressure forcing as an input, applying this assumption here simply means that we impose the IB response for the ocean rather than allowing for some dynamic response [e.g., Dickman, 1998], considering the period range of our interest, i.e., longer than 10–20 days.

5.2. Results and Contributions From Ellipticity-Induced and Topographic Torques

The resultant time series of excitations for the observed \( \phi \), the atmospheric angular momentum \( \phi_a \), their residual \( \phi - \phi_a \), and the oceanic torques \( \phi_o \), for (a) the \( x \) component and (b) the \( y \) component for 1988–1995. A quadratic trend has been removed from each series. The unit is milliarc seconds.

Figure 12. Time series of observed excitation functions of polar motion \( \phi \), the atmospheric angular momentum \( \phi_a \), their residual \( \phi - \phi_a \), and the oceanic torques \( \phi_o \), for (a) the \( x \) component and (b) the \( y \) component for 1988–1995. A quadratic trend has been removed from each series. The unit is milliarc seconds.
term of equation (10), consequently reducing the amplitude by 1/1.44 from the torque contribution itself. First, we compare the seasonal components; the result in terms of x and y components is given in Table 3. Basically, the comparison for the ellipticity-induced and the topographic torques is good in both amplitude and phase except for the semiannual y component, where virtually no signal exists in the topographic torque.

We then remove the seasonal signals from the excitation function time series. Figure 13 plots the remaining nonseasonal series. It is seen that the amplitudes of the x component are comparable, while the ellipticity torque of the y component is significantly larger than its topographic counterpart. Next, for a quantitative comparison we perform the correlation study for the nonseasonal series. We found the following:

1. For the x component the comparison result is as one would expect from geophysical consideration: The ellipticity and the topographic torques, while exhibiting no correlation between themselves, are both correlated with \( \phi - \phi_a \) (with coefficients 0.45 and 0.35, respectively), yet the correlations are weaker than that between their sum \( \phi_o \) and \( \phi - \phi_a \) (at 0.55). In other words, they collectively and independently contribute to the correlation between \( \phi_o \) and \( \phi - \phi_a \) found above.

2. For the y component, both torques have strong correlations with \( \phi - \phi_a \) (with 0.57 and 0.42, respectively) but also with each other even more (with 0.59). The reason for the latter remains unexplored. The difference in the above behavior between the x and y components is presumably related to the actual geographical distribution of the ocean on the globe.

Finally, we examine the effect of the application of the gravitational and deformational correction to the topographic torque (a multiplication factor in terms of load Love numbers), which is introduced by Wahr [1982, equation (5.17)]. We compared the time series of the nonseasonal topographic torque excitation, \( \phi_{ot} = (\phi_{ot}(x) + i \phi_{ot}(y)) \), with and without the correction. We found that the standard deviations for \( |\phi_{ot}| \) is reduced by ~10% when the correction is applied: 8.4 mas versus 9.2 mas (where mas is milliarc seconds). We also found a little increase in the correlation coefficient between \( |\phi_{ot}| \) and \( |\phi - \phi_a| \) due to the correction: 0.38 versus 0.37. Although the improvement in the correlation is little, it is reassuring to see that the gravitational and deformational correction formulated by Wahr [1982] does lead to a better explanation of the observed excitation.

### 5.3. Some Interesting Meteorological Signals

We now describe certain rather interesting features in the interannual variability during the period we studied, 1988–1995 (Figure 12), which are not elaborated by Johnson et al. [1999]. It is well known that LOD is highly correlated with El Niño-Southern Oscillation (ENSO), or the El Niño-La Niña sequence, corresponding to an anomalous variation in the axial component of AAM. Particularly, relevant to the present discussion is a prominent 1988–1989 La Niña signature in LOD series pointed out by Chao and Au [1995]. On the other hand, virtually no overall correlation exists between the observed polar motion excitation and ENSO’s proxy Southern Oscillation Index (SOI) [Chao and Zhou, 1998]. Indeed, it has long been noted that the equatorial components of AAM have little El Niño-La Niña signatures (e.g., Chao and Au, 1991).

In Figure 12a we see a prominent dip in the observed x component time series coincident in time with the strong 1988–1989 La Niña. Except possibly for a short period in early 1989, the AAM (the second curve) carries little indication of the occurrence of the La Niña. The oceanic torque series (the bottom curve), on the other hand, shows a clear signature during 1988, which apart from the inadequate amplitude mentioned above, corresponds quite well to the non-AAM excitation dip (the third curve). A similar, but somewhat less prominent, dip occurred in 1992 around a mild El Niño period, and strong fluctuations occurred in 1995. Again, these signatures are absent in AAM. A similar behavior is found in the y component, albeit not as clear cut. Thus, frequently, the oceanic torque is solely responsible for interannual fluctuations in the polar motion excitation. It is
interesting to note that the oceanic torque itself is the ocean’s response to atmospheric forcing; hence one can say that in these cases the oceanic torque acts as a proxy for the atmosphere in exciting polar motion. Detailed case studies in relation to El Niño and La Niña episodes are worth pursuing.

6. Summary

We calculated various oceanic torques that act on the solid Earth using model output of the Parallel Ocean Climate Model at 3-day intervals for the period of 1988–1995. We do so for all three dimensions, including the equatorial (x, y) components that excite polar motion; and the axial (z) component which affects LOD. We analyzed these time series, compared them, and examined their consistency. We also summarize the result of the application to the Earth rotation study based on our torque calculations.

Table 2 summarizes the oceanic torques in a qualitative physics setting. Major conclusions are as follows: (1) Total ellipticity-induced torque (which is the algebraic sum of the pressure torque due to the ellipticity gradient and the gravitational torque) has the highest power in the x and y components and vanishes in the z component. (2) Topographic torque (pressure torque due to ocean bottom topography) is second most powerful. (3) Ocean bottom frictional torque is negligible compared with the other torques. (4) The effective torque derived from the mass terms of the equatorial OAM coincides almost perfectly with the ellipticity-induced torque. (5) The effective torque derived from the motion term of the equatorial OAM can be explained well by the topographic torque except for the seasonal signals. (6) The effective torque derived from the axial OAM is near zero, implying a direct transfer of wind frictional torque to the ocean bottom topographic torque. (7) Both the ellipticity and topographic torques contribute to the excitation of polar motion at a similar level for both seasonal and nonseasonal variations. (8) Some meteorological signals coincident to the El Niño and/or La Niña episodes show up in the oceanic excitation rather than the atmospheric excitation of the polar motion.

While it appears to have some disadvantages relative to the angular momentum approach, the torque approach does treat different physical interaction sources separately. This paper quantitatively demonstrates the characteristics and validity of the oceanic torque calculations, indicating that the torque approach based on general circulation model outputs can reveal the oceanic contributions to the polar motion excitation. Further refinements of the ocean torque approach in Earth rotation dynamics await future studies.

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