Global gravitational changes due to atmospheric mass redistribution as observed by the Lageos nodal residual

Benjamin Fong Chao\textsuperscript{1} and Richard Eanes\textsuperscript{2}

\textsuperscript{1} Geodynamics Branch, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA
\textsuperscript{2} Center for Space Research, University of Texas, Austin, TX 78712, USA

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**SUMMARY**

Variations in the even-degree zonal gravitational field will produce perturbations to the otherwise constant nodal precession rate in satellite orbits. The contribution of the atmospheric mass redistribution to this nodal ‘excitation’ on the laser-ranging geodetic satellite Lageos is computed using the ECMWF (European Centre for Medium-range Weather Forecasts) meteorological analysis data over the period 1985–1991, with and without assuming the inverted-barometer (IB) effect for the ocean. These excitation time series are then compared with the observed Lageos nodal residual caused by global gravitational variations. Three frequency bands are examined with the following results. (i) In the interannual band (longer than a year), an apparent $\sim$14-month phase lead is found in the atmospheric excitation relative to the Lageos observation. (ii) At seasonal periods, the atmospheric annual signal agrees reasonably well with the observations; the agreement is further strengthened by incorporating continental hydrological contributions. The semi-annual signal compares poorly with observations, indicating the presence of other geophysical sources than the atmosphere. (iii) At the intraseasonal time-scale (shorter than a year) a wide-band correlation coefficient of 0.64 is found between the observed and the atmospheric excitations with the IB effect. The non-IB model yields a somewhat lower correlation of 0.59, but a better correspondence in amplitude. The correlations, together with the corresponding coherence spectra, clearly demonstrate a strong atmospheric contribution to the global gravitational variations.

**Key words:** atmospheric mass, gravitational change, Lageos' node.

**1 INTRODUCTION**

The external gravitational potential field of a massive body is determined by the internal mass distribution of the body, governed by the Newtonian gravitational law. When the mass distribution varies with time, the gravitational field varies accordingly.

Since the beginning of the space age, orbit determination for geodetic satellites has proved an effective means and provided a wealth of observational data for determining the Earth’s external gravitational potential field $U$. The technique of satellite laser ranging (SLR) in particular has been instrumental, the most notable case being the Lageos satellite which has continued to provide high-precision data since its launch in 1976. In recent years, improvements in the SLR technique and in the modelling of Lageos' orbit have reached the point where some minute orbital perturbations can be detected to reveal temporal variations in the low-degree zonal harmonics of $U$ (Cheng et al. 1989; Nerem et al. 1993; Gegout & Cazenave 1993). In particular, Lageos’ nodal residual has been a most readily measurable orbital parameter. In this paper the time rate of the nodal residual is used as a proxy for the global gravitational variations. As a function of time, it represents a linear combination of the variations of low-degree even zonal harmonics of $U$.

What are the geophysical phenomena that can cause global mass redistributions, and hence the gravitational variations? There is no shortage of candidates (see Chao 1993b for a review). The tidal deformation in the solid earth and ocean is now routinely modelled in satellite orbit determination, although the modelling of some oceanic tides is still problematic. Large-scale hydrological variations exchange water mass between the ocean and the continents. Examples include rain and snow, the mass balance in polar ice sheets and underground water storage, the denudation of
mountain glaciers, and water impoundment in artificial reservoirs. Thermal-haline and wind-driven circulations in the ocean also redistribute a great quantity of mass. In the solid earth, seismic mass displacement, the post-glacial rebound, mantle convection, and core activities all contribute to global gravitational changes. However, one of the largest non-tidal contributions to global gravitational variation felt at orbital distances comes from mass transport in the atmosphere; that is the subject of this paper.

The atmospheric mass distribution varies mostly on a temporal scale ranging from interannual to as rapid as a few days, with strong seasonal power. The mass ratio of atmosphere to the Earth is about 10^-9, and the relative root-mean-square (rms) fluctuation of pressure at global scales is typically of the order 10^-3. This results in \( \sim 10^{-9} \) rms variations in \( U \), manifested particularly in the low-degree harmonics now detectable by the SLR technique.

The first evidence of non-tidal gravitational variation was found in Lageos' secular nodal acceleration (Yoder et al. 1983; Rubincam 1984), and was attributed to the post-glacial rebound of the mantle. Atmospheric influence primarily at seasonal periods was later proposed by Gutierrez & Wilson (1987) based on the conventional climatological data available at the time, as compared with nodal acceleration of Lageos and Starlette, another laser-ranging satellite. Similar results were reported by Cheng et al. (1989). Chao & Au (1991) presented computations employing modern meteorological analysis data and compared them with Lageos and Starlette observations. Recently, Nerem et al. (1993) and Gegout & Cazenave (1993) reported a comparison of similar atmospheric computations with Lageos observations at monthly intervals. These works represent a progression in the quest for higher temporal resolutions. The present paper continues this legacy. Our procedure consists of (i) computing the effect of the atmospheric mass movement on Lageos' node, and (ii) comparing it with the observations at 5-day intervals in terms of seasonal as well as intraseasonal, broad-band signals. The time span studied is approximately 6.4 years, from 1985 to 1991.

An interesting parallel can be drawn with respect to the progression of the research of Earth rotation excitations. Although governed by a completely independent physical principle (namely the conservation of angular momentum), the mass redistribution mechanisms mentioned above would lead to similar changes in the Earth's rotation, in both length-of-day and polar motion. Among them, the atmospheric angular momentum variation is known to be the dominant source for length-of-day variations shorter than several years (e.g. Hide & Dickey 1991). The same variation is also largely responsible for the excitation of the annual wobble and more rapid polar motion (e.g. Chao 1993a). The history of this research has seen a decisive, progressive shift of emphasis towards ever higher temporal resolutions, as more high-quality data become available in both space geodesy and meteorology.

### 2 Theory

The Earth's external gravitational field \( U(r) \) at position \( r = (r, \theta, \lambda) \) (where \( r \) is the radial distance, \( \theta \) the co-latitude and \( \lambda \) the east longitude) is customarily expressed in a harmonic multipole expansion in terms of the dimensionless Stokes coefficients (e.g. Kaula 1966):

\[
U(r) = \frac{GM}{r} \left[ 1 + \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left( \frac{R}{r} \right)^l P_{lm} \cos \theta \right] \times \left( C_{lm} \cos m\lambda + S_{lm} \sin m\lambda \right).
\]

In (1), \( G \) is the gravitational constant, \( M \) and \( R \) are the Earth's total mass and mean radius; \( P_{lm} \) is the 4\( \pi \)-normalized associated Legendre function of degree \( l \) and order \( m \). The Stokes coefficients \( C_{lm} \) and \( S_{lm} \) are directly proportional to the multiple moments of the Earth's mass distribution (e.g. Chao & Gross 1987). In particular,

\[
J_l = -\frac{1}{MR^2} \int \rho(r)r^2 P_l(\cos \theta) \, dV,
\]

where the (un-normalized) zonal \('J'\) coefficient of degree \( l \) is defined as \( J_l = -(2l+1)^{1/2}C_{l0} \), \( \rho(r) \) is the mass density of the Earth, \( P_l \) is the ordinary Legendre polynomial, and the integration is over the entire volume of the Earth (including the atmosphere) with volume element \( dV = r^2 \sin \theta \, dr \, d\theta \, d\lambda \).

Any temporal variation in \( \rho(r) \) will, according to (2), give rise to a temporal variation in \( J_l \), or \( \Delta J_l(t) \). For the Earth, the \( \Delta J_l(t) \)s are generally no larger than \( 10^{-9} \). For mass redistributions in the atmosphere (where \( r = R \)) in a Eulerian description and assuming a vertical hydrostatic profile for the atmosphere, it can be shown that

\[
\Delta J_l(t) = -\frac{1+k_l'}{Mg} R^2 \int \Delta \rho(\theta, \lambda, t) P_l(\cos \theta) \sin \theta \, d\theta \, d\lambda,
\]

where \( g \) is the average gravitational acceleration, and \( \Delta \rho \) is the departure of the surface air pressure from the local mean state. The Earth's load Love numbers, \( k' \), have been computed by Farrell (1972); \( k'_l = -0.31 \), \( k'_1 = -0.20 \), \( k'_0 = -0.13 \), etc. The factor \((1+k'_1)/1.5\) accounts for the elastic yielding effect of the solid Earth under surface loading/unloading (Munk & MacDonald 1960).

The response of the fluid ocean to the overlying atmospheric loading/unloading should also be properly treated in the evaluation of (3). The extreme case where the ocean can react isostatically in the fashion of an inverted barometer will be referred to as IB. As a result of IB, \( \Delta \rho \) over the ocean will be spread uniformly and instantaneously over the entire oceanic area, greatly reducing the net gravitational effect. The other idealized extreme, called non-IB, simply assumes a rigid ocean which would ignore any overlying pressure change. The extent to which the IB effect is valid in reality (presumably as a function of the temporal and spatial scales in question) remains an outstanding problem. In this paper we will do the computation both ways, IB and non-IB (cf. Chao & Au 1991; Chao 1993a), and both results will be compared with observations.

In the Newtonian gravitation, the orbit of an Earth-orbiting satellite would be invariant (closed ellipse) when \( \rho(r) \), and hence \( U \), is spherically symmetric. In general, non-sphericity in \( \rho(r) \) will cause the orbit to deviate. For example, under the influence of even-degree zonal harmonics in \( U \) (dominated by \( J_2 \) which, of course, is by far
the largest term in $U$ next to $GM$), the orbital plane would undergo a nodal precession in which its node (defined as the point of intersection of the orbit with the Equatorial plane) drifts slowly along the Equator. The nodal precession is retrograde (or prograde) for satellites with orbit inclination smaller (or greater) than 90\degree. If these harmonics are constant in time, the satellite's nodal precession will also be held at a constant rate. When they vary (only slightly for the Earth due to geophysical mass redistributions), the nodal precession rate varies accordingly. The angular departure of the true node from the nominal node (which is precessing at a constant rate computed from some standard static model of $U$) is referred to as the node residual. There are other orbital fluctuations that are potentially useful, for example that in the orbit eccentricity bears information about the odd-degree zonal harmonics. Unfortunately, their determination is presently less reliable because of modelling uncertainties.

Lageos' node residual is most readily subject to SLR measurement. The parameter (used in this paper) to be directly compared with geophysical mass variations is the nodal excitation, $\Delta \omega$, defined as the time rate of the node residual and designated by $\sigma$. For Lageos, $\sigma$ (in units of milliarcsecond per year, or mas yr$^{-1}$) is given in terms of $\Delta \omega$ (degree $l = 2, 4, 6, \ldots$, in units of $10^{-10}$) as

$$\sigma = 41.6 \Delta \omega_2 + 15.4 \Delta \omega_4 + 3.3 \Delta \omega_6 + 0.2 \Delta \omega_8 - 0.1 \Delta \omega_{10} - 0.1 \Delta \omega_{12}$$

(Cheng et al. 1989). The contribution of the harmonics to the nodal precession decreases with $l$, depending on the altitude of the satellite (Kaula 1966). The decrease is fairly rapid and becomes negligible after degree 6 for Lageos owing to its high altitude. We will use the time series $\sigma(t)$ as a proxy for the global gravitational variations. Note, however, that the contributions from individual even $J$s are inseparable with node observations from only one satellite.

3 ATMOSPHERE AND LAGEOS DATA

The global surface air pressure data needed for the evaluation of eq. (3) are adopted from the meteorological analysis data from the European Centre for Medium Range Weather Forecasts (ECMWF). They are derived for the mean elevation (0 m for the sea-level) at each grid-point, originally given on a 2.5° latitude by 2.5° longitude grid. The data set that we use has been reprocessed and interpolated onto a 2° latitude by 2.5° longitude grid, as described by Schubert et al. (1990). The data are given twice daily at 0000 UT (Greenwich midnight) and 1200 UT (Greenwich noon). Details of how the ECMWF data are used in the present computation can be found in Chao & Au (1991).

In this paper we choose to analyse a total time span of 6.4 years: from 1985 May 1 to 1991 September 16, to avoid undesirable data non-uniformities in the ECMWF series. Although the ECMWF data have been available since 1980, over the years ECMWF has had several modifications in the circulation models used for the prediction and analysis of data. These changes usually result in systematic errors in the form of discontinuities in the time series. The severity of the discontinuity depends on the parameter in question and the nature of the model change. One major change, which is evident particularly in the total mass of the atmosphere, was implemented on 1985 May 1. The data set remains uniform until 1991 September 17, when a further similar change was implemented.

The integration (3) over the geographic grid is performed using a linear interpolation scheme, i.e. the trapezoidal rule. The computed $\sigma$, both for IB and non-IB, are substituted into (4) to yield the predicted Lageos nodal excitations due to the atmospheric variation. Designated as $\sigma_a[\text{IB}]$ and $\sigma_a[\text{non-IB}]$, they are shown as the top two curves in Fig. 1.

Lageos was launched on 1976 May 4 into a near-circular orbit with a semi-major axis of 12 770 km and inclination of 109.9°. The nominal constant nodal precession rate is about $0.343$ d$^{-1}$, or $4.51 \times 10^8$ mas yr$^{-1}$. Moderate smoothing was applied during the orbit determination procedure to the long-arc (16 yr) solution of the node residuals at 5-day intervals; the effective cut-off period is roughly 20–30 days. The observed nodal excitation series $\sigma_a(t)$ is obtained by time-differentiating the node residual series. The node residual is with respect to a nominal node model which actually includes a nominal ocean tide model from Schwiderski (1980). Errors associated with the latter thus permeate into the $\sigma_a$ solution, but only at specific aliased tidal periods as seen from (the slowly precessing) Lageos, typically at several hundred days (e.g. Yoder et al. 1983). A more detailed description is given in Cheng et al. (1989). The Lageos-observed nodal excitation series $\sigma_a(t)$ is shown at the bottom of Fig. 1.

4 RESULTS AND DISCUSSION

The task now is to compare quantitatively $\sigma_a[\text{IB}]$ and $\sigma_a[\text{non-IB}]$ with $\sigma_a$ in order to reveal the atmospheric contribution to the total observed gravitational variation. Our numerical methodology parallels that of Chao (1993a) in a polar-motion excitation study.

The atmospheric $\sigma_a$ series contains a great deal of high-frequency information which fortunately is absent from the Lageos observation $\sigma_a$ (see Fig. 1). Some pre-processing of the $\sigma_a$ series is thus necessary in order to proceed with the comparison. First, $\sigma_a$ are low-pass filtered (with a Chebyshev type I filter) and then decimated by a factor of 10 to match the 5-day sampling interval of $\sigma_a$. They are then passed through a symmetric 5-point running mean filter to match spectrally the procedure applied to the Lageos data. Fig. 2 shows an overlay of the results (solid line) with the $\sigma_a$ series (dotted line). Some general agreement is evident.

Three frequency bands will be treated separately in the study below: interannual, seasonal and intraseasonal.

4.1 Interannual variation

We shall extract the interannual as well as the seasonal signals simultaneously by performing a (linear) least-squares fit of the time series to a sum of a fourth-degree polynomial plus two sinusoids (sine and cosine) for the annual signal and two sinusoids (sine and cosine) for the semi-annual signal. The resultant polynomial function is an optimal representation of the interannual variations during the study period. The polynomial degree of 4 is chosen empirically:
lower degrees seem insufficient to represent fully the slow interannual variations, while higher degrees tend to yield instability near the end points of the time series. The interannual variations are compared in Fig. 3. During 1985–1991 the interannual variations all undergo a similar full 'cycle'. The interannual amplitude of \( \sigma_o \) (IB) is comparable to that of \( \sigma_o \) (non-IB) (Fig. 4), and they both lead \( \sigma_a \) in phase by about 14 months. Based on the relatively short time span of 6.4 yr, it is probably premature to draw geophysical conclusions from this comparison. Nevertheless, it is interesting to note that the delayed oscillator model of Schopf & Suarez (1988) would predict an oscillation in the equatorial Pacific ocean basin with a characteristic period of about 14 months under certain conditions (Philander 1990). A more complete study awaits longer time series.

### 4.2 Seasonal variations

The least-squares solutions for the two pairs of seasonal sinusoids described above constitute the seasonal variations.

In this paper we represent a pair of sinusoids (sine and cosine) by its amplitude \( A \) and a phase angle \( \phi \) defined in the sine convention relative to January 1.0: \( A \sin (\omega t + \phi) \), where \( t = 0 \) refers to January 1.0. An advantage of this definition is that polar quantities (\( A, \phi \)) add like vectors and the argument \( \omega t + \phi \) traverses with time in the 'natural' counterclockwise sense (Chao \& O'Connor 1988).

The resultant annual excitation are listed in Table 1 and plotted as 'vectors' in Fig. 4. The atmospheric \( \sigma_o \) (non-IB) vector compares well in amplitude with the Lageos-observed \( \sigma_o \) vector but differs considerably in phase, whereas the opposite is true for the atmospheric \( \sigma_o \) (IB) vector. Table 1 and Fig. 4 also give the Lageos' nodal excitation predicted using the annual \( \Delta J_2 \) and \( \Delta J_4 \) computed by Chao \& O'Connor (1988) for the global redistribution of continental snow and rain. They are labelled 'hydrological'. As is evident in Fig. 4, the addition of this hydrological contribution to the atmospheric contribution leads to a significantly better agreement for the 'sum' vectors with \( \sigma_o \), strengthening the conclusion of a previous study by Chao \& Au (1991). This implies that other contributions (such as ocean circulation) are probably secondary at the annual period. It is particularly interesting to note that the \( \sigma_o \) vector lies between the extreme cases of \( \text{sum}[\text{IB}] \) and \( \text{sum}[\text{non-IB}] \), both in amplitude and in phase. This is consistent with the notion that the ocean behaves in a manner between the IB and non-IB idealizations.

The semi-annual terms, on the other hand, compare poorly: the atmospheric excitations and the estimated hydrological excitation are all much too small to account for the Lageos-observed data. The phases appear to be unrelated too. This signifies the importance of effects from other geophysical sources, such as ocean circulation, at the semi-annual period. Although the 'sum' vectors are given in Table 1, it should be noted that the atmospheric and the hydrological semi-annual excitations are roughly opposite in phase. Therefore the sum vectors are prone to errors in both estimates. These results are consistent with the findings of Chao \& Au (1991) and Nerem et al. (1993).

### 4.3 Intraseasonal variations

We now subtract the interannual as well as the seasonal variations from the nodal excitation series. The results represent the broad-band intraseasonal variations, and are
compared in Fig. 5. Again, it is interesting to note that the $\sigma_\alpha[\text{IB}]$ series tends to 'under-represent' $\sigma_\alpha$, whereas $\sigma_\alpha[\text{non-IB}]$ does the opposite, 'over-representing' $\sigma_\alpha$.

The nodal excitation series in Fig. 5 are then subject to two (complementary) correlation analyses: the time-domain correlation function, which gives an overall correlation estimate as a function of relative time lag, and the frequency-domain complex coherence spectrum, which provides a spectral decomposition of the overall correlation.

Figure 6 shows the correlation magnitude versus the time lag of $\sigma_\alpha$ relative to $\sigma_\alpha[\text{IB}]$ and $\sigma_\alpha[\text{non-IB}]$. The correlation around the zero lag time is striking (in contrast to the
Table 1. Comparison of the (Amplitude, Phase)* estimates for the seasonal (annual and semi-annual) nodal excitation for Lageos.

<table>
<thead>
<tr>
<th></th>
<th>Annual</th>
<th>Semiannual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lageos-observed $\sigma_o$</td>
<td>(108.2, -156°)</td>
<td>(86.1, -35°)</td>
</tr>
<tr>
<td>Atmospheric $\sigma_o$[IB]</td>
<td>(44.9, -160°)</td>
<td>(10.8, -86°)</td>
</tr>
<tr>
<td>Atmospheric $\sigma_o$[non-IB]</td>
<td>(101.5, -180°)</td>
<td>(19.4, -62°)</td>
</tr>
<tr>
<td>Hydrological</td>
<td>(59.5, -121°)</td>
<td>(24.1, 86°)</td>
</tr>
<tr>
<td>Sum[IB]</td>
<td>(98.5, -137°)</td>
<td>(13.5, 80°)</td>
</tr>
<tr>
<td>Sum[non-IB]</td>
<td>(141.4, -159°)</td>
<td>(12.8, 33°)</td>
</tr>
</tbody>
</table>

* Amplitude is in units of mas yr$^{-1}$, Phase is with respect to January 1.0 in the sine convention.

The finding of Gegout & Cazenave (1993). It reaches a coefficient of 0.64 for IB and 0.59 for non-IB. The interpolated maximum occurs somewhere between 0 to 5 days of time lag; the significance of this time lag is difficult to quantify as its face value is shorter than the resolution of the $\sigma_o$ series of 5 days.

Thus, the IB effect yields a higher correlation estimate than the non-IB case, and the difference is statistically significant given the large number of observations. Note that the correlation function, being a normalized measure of the waveform pattern only, conveys no information about the relative amplitude. Two observations are then worth noting. First, the two $\sigma_o$ curves (IB and non-IB) are highly correlated between themselves (they show very similar waveforms in Fig. 5 although their amplitudes differ considerably). This is reflected in the similarity in the shapes of the two correlation functions with $\sigma_o$ (Fig. 6). Secondly, it is evident from Fig. 5 that, in terms of the relative amplitude, it is often the non-IB model that appears to yield better visual correspondence with $\sigma_o$. Combined with the above finding for the correlation coefficients, this apparently again implies that the reality is somewhere between IB and non-IB. The same phenomenon was consistently found by Nerem et al. (1993) in a similar study, and by Chao (1993a) in a polar-motion excitation study.

Figure 7 shows the coherence spectra of $\sigma_o$ with $\sigma_o$[IB] and $\sigma_o$[non-IB] in terms of the coherence magnitude squared ($\Gamma^2$) and coherence phase. Here we used the multitaper technique of Thomson (1982) to evaluate the cross- and auto-spectra in computing the coherence. This technique provides robust and minimum-leakage spectral estimates: Seven orthogonal tapers with a time-bandwidth product of 4π were adopted (e.g. Park, Lindberg & Vernon 1987). To estimate the $\Gamma^2$ threshold at a given percentage confidence level $\alpha$, we run a Monte Carlo simulation in a similar manner to that in Chao (1988). The result indeed matches well with $M = 7$ degrees of freedom using analytical relation usually given in textbooks (e.g. Bloomfield 1976): $\Gamma^2(\alpha) = 1 - (1 - \alpha)^{1/(M-1)}$. Some key values are: $\Gamma^2(90 \text{ per cent}) = 0.32$, $\Gamma^2(95 \text{ per cent}) = 0.40$, $\Gamma^2(99 \text{ per cent}) = 0.54$, and $\Gamma^2(99.5 \text{ per cent}) = 0.59$. Notice the high coherence concentrated in the frequency band of about 2–7 cycles per year (cpy), with the corresponding phase spectrum at near zero. A lesser coherence occurs at about 27 cpy (about a fortnightly period); the significance of this correlation is uncertain as the power associated with it is rather low.

It is also observed that the higher correlation for IB is primarily concentrated in the low-frequency band of 2–7 cpy. Within the intraseasonal band, this is consistent with the notion that the longer period is more favourable for the IB effect to take place. Future research incorporating realistic ocean models should lead to a more accurate assessment of the role played by the atmosphere and oceans in changing the Earth’s gravitational field. At present, we can conclude that the atmospheric mass redistribution is a
primary source for global gravitational variations on the intraseasonal time-scale, particularly at longer than 2 months but perhaps down to sub-monthly time-scales.

Finally, we should point out that the reported research (i.e. geophysical excitation of global gravitational variations) is still in its infancy, rather reminiscent of the status of the corresponding study of Earth rotation variations 10–15 years ago. The quality of the gravitational observations (in both precision and accuracy) is only marginally adequate, and our modelling of satellite orbital dynamics need further improvements. More extensive laser-ranging coverage to multiple satellites can lead to higher accuracy and higher temporal resolution for the gravitational changes. It can also lead to some, but still limited, spatial resolution as a result of the separability of the gravitational harmonics such as those in eq. (4). A complete solution, however, would...
Figure 5. As Fig. 2, except that the interannual and the seasonal (annual and semi-annual) variations have been removed from the series, showing comparison for the intraseasonal variations.
Figure 6. The cross-correlation function of the intraseasonal series in Fig. 5 during 1985–1991: (a) between $\sigma_o$ and $\sigma_o\text{[IB]}$; (b) between $\sigma_o$ and $\sigma_o\text{[non-IB]}$.

Figure 7. The squared magnitude and phase of the (multitapered) complex coherence spectra for the intraseasonal series in Fig. 5 during 1985–1991: (a) between $\sigma_o$ and $\sigma_o\text{[IB]}$; (b) between $\sigma_o$ and $\sigma_o\text{[non-IB]}$. 
require new technology that provided much higher observational sensitivity as well as much higher temporal and spatial resolutions (e.g. Colombo & Chao 1992).

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