Abstract. Besides generating seismic waves, which eventually dissipate, an earthquake also generates a static displacement field everywhere within the Earth. This global displacement field rearranges the Earth’s mass distribution, causing the Earth’s rotational properties and gravitational field to change. The size of these changes depends, in general, upon the size of the earthquake. The Macquarie Ridge earthquake of May 23, 1989 is considered to be the largest earthquake to have occurred since the 1977 Sumba and Tonga events. As such, the coseismic effect of this earthquake upon the Earth’s length-of-day, polar motion, and low-degree harmonic coefficients of the gravitational field are computed. It is found that this earthquake should have caused the length-of-day to decrease by 0.06 μsec, the position of the mean rotation pole to shift 0.11 milli-arcsec towards 323 °E longitude, and selected degree l = 2-5 gravitational field coefficients to change by about 1 part in 1013. These changes are all smaller than can be detected by current observational techniques. However, changes of this size could perhaps be detected in the future with the implementation of proposed improvements to the techniques of monitoring the Earth’s rotation, and (especially for the low-degree gravitational field coefficients) with the placement of GPS receivers onboard orbiting spacecraft.

Introduction

The deformation field generated by an earthquake is, of course, largest near its source. However, it is not insignificant even at teleseismic distances from the source [Press, 1965]. Thus the displacements experienced by mass elements of the Earth far from the earthquake source can, when integrated over the entire Earth, contribute significantly to changes in global geodynamic properties of the Earth such as its rotation and gravitational field. The importance of this far-field displacement field upon the Earth’s rotational properties has been recognized by a number of investigators concerned with evaluating earthquakes as a possible excitation source of the Chandler wobble [e.g., Dahle, 1971, 1973; Gross, 1986]. Changes in surface gravity have been observed following large earthquakes [e.g., Barnes, 1966], and a number of studies have been done to predict such changes based upon an elastic dislocation model of the earthquake [e.g., Rundle, 1979; Walsh and Rice, 1979; Savage, 1984]. On a global scale, Chao and Gross [1987] have computed the effect of earthquakes on the spherical harmonic coefficients of the Earth’s gravitational field.

The magnitude of the displacement field generated by an earthquake is proportional to the energy released by it. In general, larger earthquakes produce greater displacement fields which have greater effects upon the Earth’s rotational properties and gravitational field. The Macquarie Ridge earthquake of May 23, 1989 (Ms = 8.2) is thought to be the largest earthquake to have occurred since the 1977 Sumba and Tonga events. This study is undertaken in order to evaluate the effect of this event upon global geodynamic properties of the Earth. In particular, the theoretical coseismic effect of the Macquarie Ridge earthquake upon the Earth’s rotational properties and low-degree spherical harmonic coefficients of its gravitational field are computed. Observations of the Earth’s rotational properties are examined for the expected signatures, and the magnitude of the expected changes in the gravitational field coefficients are compared to the precision with which they are being currently determined from satellite tracking data. It should be emphasized that this paper is concerned with just the coseismic effect of the Macquarie Ridge earthquake. In general there can be pre-seismic and post-seismic motions associated with an earthquake that can contribute to changing the Earth’s rotational properties and gravitational field. However, the modelling of these effects is beyond the scope of the present paper, as are the effects of viscoelasticity.

Theory

The procedure described in detail by Chao and Gross [1987], and summarized below, for modeling the effects of earthquakes on the Earth’s rotation and gravitational field is followed here.

Gravitational Field Changes

The gravitational potential $U(r)$ at some external field point $r = (r, \theta, \phi)$ due to a body of arbitrary shape and density distribution $\rho(r)$ is given by:

$$U(r) = G \int \frac{\rho(r)}{|r - r_o|} \, dV$$

where $G$ is the gravitational constant, and the volume integral extends over the entire volume $V_o$ of the body. By expanding in terms of spherical harmonics, the inverse distance between the external field point and the position of some mass element located initially at $r = (r, \theta, \phi)$, the gravitational potential (1) can be written as:

$$U(r) = 4\pi G \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y^m_l(\theta, \phi) \int r^l Y^m_l(\theta, \phi) \, \rho(r) \, dV$$

where the $Y^m_l(\theta, \phi)$ are the fully normalized surface spherical harmonic functions of degree $l$ and order $m$. However, in geodesy the gravitational potential is usually given in the form [e.g., Kaula, 1966]:

$$U(r) = \frac{GM}{r_o} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \left( \frac{r}{r_o} \right)^l C_{lm} \cos m\phi_o + S_{lm} \sin m\phi_o$$

where the dimensionless, normalized, real-valued $C_{lm}$ and $S_{lm}$
are known as the Stokes coefficients, $a$ is some reference radius (less than $r_o$ and later taken to be the radius of the spherically symmetric Earth model), and the $P_{lm}$ are the normalized associated Legendre functions. From the equivalence of expressions (2) and (3), it is straightforward to show that the Stokes coefficients are simply the normalized multiple moments of the density field $\rho(r)$:

$$C_{lm} + i S_{lm} = \frac{N_{lm}}{M a^l} \int_{V_h} r^l Y_{lm}(\theta, \phi) \rho(r) \, dV$$  (4)

for $0 \leq m \leq l$ where $N_{lm}$ is a normalization constant. Note that since $Y_{lm}(\theta, \phi) \propto \exp(ilm\phi)$, which is real-valued for $m = 0$, all $S_{0m} = 0$. Furthermore, $C_{00}$ is a constant; and if the origin of the reference frame is placed at the center-of-mass of the body, then all $C_{lm}$ and $S_{lm}$ terms are zero. Thus the non-trivial terms start with degree $l = 2$.

When an earthquake occurs, the Earth's density distribution is perturbed, causing the Stokes coefficients to change. In order to compute this change, a Lagrangian approach is taken wherein a mass element $dM$ is followed during the deformation process from its initial position to its final position $r + u$. If the displacements due to the deformation are small, then the integrand in expression (4) for the Stokes coefficients can be expanded in a Taylor series yielding:

$$\Delta C_{lm} + i \Delta S_{lm} = \frac{N_{lm}}{M a^l} \int_{V_h} r^{l-1} u \cdot \nabla Y_{lm}(\theta, \phi) \rho(r) \, dV$$  (5)

for their first-order change where the integration is over the initial, undeformed volume of the Earth model. The quantity in parentheses is an operator operating on $Y_{lm}(\theta, \phi)$ where $\nabla_h$ is the horizontal gradient operator.

Expression (5) relates the change in the Stokes coefficients to some specified displacement field $u(r)$. In the present study the displacement field of interest is that of the mass field generated by an earthquake. An expression for this field can be obtained by seeking a normal mode solution of the equations of motion. That is, the (in general) time-dependent displacement field $u(r, t)$ is expanded in a complete set of normal modes of some Earth model where the static, i.e. at time $t = \infty$, expansion coefficients $a_k(\infty)$ can be shown to be [e.g., Gilbert, 1970; Gilbert and Dziiewonski, 1975]:

$$a_k(\infty) = \frac{1}{\alpha_k} M E_k(r_o).$$  (6)

The eigenfrequency of the $k$'th normal mode is $\omega_k$, $M$ is the moment tensor of the earthquake specifying its source properties, and $E_k(r_o)$ is the strain tensor associated with the $k$'th normal mode evaluated at the centroid $r_o$ of the source volume. A spherically symmetric, non-rotating, elastic, isotropic (SNREI) Earth model has two classes of normal modes, namely, the spheroidal and the toroidal. Since the spheroidal eigenvectors $\sigma_l^m$ are orthogonal to both $V_h$,$Y_{lm}(\theta, \phi)$ and $\nabla_h$, its dot product in (5) will be zero and the toroidal modes can be written:

$$\Delta C_{lm} + i \Delta S_{lm} = \frac{N_{lm}}{M a^l} \int_{V_h} r^{l-1} u \cdot \nabla_h Y_{lm}(\theta, \phi) \rho(r) \, dV$$  (7)

where the subscript $n$ denotes the radial overtone number. The final expression for the earthquake-induced coseismic changes of the Stokes coefficients is obtained by combining (5–7):

$$\Delta C_{lm} + i \Delta S_{lm} = \frac{N_{lm}}{M a^l} \sum_n M_n \frac{E_n(r_o)}{(\omega_n)^2} \int_0^{r_o} \left[ r^l \{ u(r) \} + (l+1) \nabla u(r) \} \right] \, dr$$  (8)

Earth Rotation and Orientation Changes

When an earthquake occurs, the density distribution of the Earth is perturbed causing perturbations to the inertia tensor which, by conservation of angular momentum, causes perturbations $\Delta \omega$ to the angular velocity. The linearized conservation of angular momentum equation within some Conventional Terrestrial Reference Frame can be written [e.g., Munk and MacDonald, 1960; Wahr, 1982]:

$$m + \frac{i}{\sigma_e} \dot{m} = \psi$$  (9)

$$\dot{\vec{m}} = \vec{\psi}$$  (10)

where the dot denotes time differentiation, $\sigma_e$ is the complex-valued frequency of the Chandler wobble and $m = m_r + i m_i$. The excitation functions $\psi_r$ and $\psi_i = \psi_r + i \psi_i$ are functions of the mass redistribution and associated motion induced by the earthquake, and the dimensionless quantities $m_2$ are defined by $\dot{\omega} = \Omega (m_2, m_r, m_i)$.

Expression (9) governs the motion of the rotation pole. Its solution shows that the excitation function $\psi(t)$ is that point in the complex plane about which the rotation pole is instantaneously revolving. In other words, the excitation function is the mean position of the rotation pole.

If the earthquake is assumed to occur instantaneously, then it can be shown [e.g., Wahr, 1982; Gross, 1986] that:

$$\psi = \frac{1.61}{I_{xx} - I_{yy}} (\Delta I_{xx} + i \Delta I_{yy})$$  (11)

where $I_{xx}$ and $I_{yy}$ are components of the initial inertia tensor, and $\Delta I_{xx}$ and $\Delta I_{yy}$ are perturbations to it due to the static displacement field generated by the earthquake. From the definition of the inertia tensor it is straightforward to show that these components of it are simply related to the degree $l = 2$, order $m = 1$ Stokes coefficients [e.g., Chao and Gross, 1987]:

$$\Delta I_{xx} + i \Delta I_{yy} = - \sqrt{5/3} M a^2 (\Delta C_{21} + i \Delta S_{21})$$  (12)

Thus the earthquake-induced changes in $C_{21}$ and $S_{21}$ as computed in the previous sub-section also allow the evaluation of the coseismic effect of earthquakes upon the rotation pole's mean position.

Expression (10) relates changes in the axial component of the rotation vector to its excitation function. For instantaneously-occurring earthquakes it can be shown [e.g., Wahr, 1982] that the earthquake-induced change in the length-of-day $\Delta \Lambda$ is:

$$\Delta \Lambda = \frac{L_o}{I_{zz}^{1/2}} \Delta I_{zz}$$  (13)

where $L_o$ is the nominal length-of-day of 86400 seconds duration, and $I_{zz}$ is the z-z component of the inertia tensor of just the crust and mantle of the Earth model.

In order to evaluate (13), the earthquake-induced perturbation $\Delta I_{zz}$ to the inertia tensor must be computed. For the earthquake-induced changes to polar motion the pertinent perturbations to the inertia tensor are simply related to perturbations to the Stokes coefficients. However, the $z$-component of the inertia tensor cannot be recovered from any linear combination of the five degree $l = 2$ Stokes coefficients. Chao and Gross [1987] overcome this by additionally computing the earthquake-induced change to the trace of the inertia tensor $\Delta T$, thereby allowing $\Delta I_{zz}$ to be recovered from $\Delta T$ and $\Delta C_{20}$. This procedure is followed herein.
Results and Discussion

In order to compute the effect of the Macquarie Ridge earthquake upon the Earth's rotational properties and gravitational field the normal mode eigenelements of some Earth model are needed, as well as a description of the source properties of the earthquake. As in Chao and Gross (1987), the normal modes of Earth model 1066B (Gilbert and Diewonki, 1975) are used for the normal mode eigenelements. The centroid-moment tensor solution obtained by the Harvard group (G. Zwart, personal communication, 1989) for the Macquarie Ridge earthquake is used as a description of its source properties. In order to place the results obtained for this earthquake in context with those of other recent large events the results for the 1960 Chile, 1964 Alaska, and 1977 Sumba events, which have been previously reported by Chao and Gross (1987), are also presented here.

Table 1 presents the estimated coseismic effect of the Macquarie Ridge earthquake upon the rotational properties of the Earth. It is found that this earthquake should have decreased the length-of-day by 0.06 µsec and shifted the mean position of the rotation pole by 0.11 milli-arcsec (equivalent to 3 mm at the Earth's surface) towards 323 øE. longitude. With the present-day space-geodetic observational techniques of satellite laser ranging and very long baseline interferometry, Universal Time (UT1) can be determined to within about 0.1 msec and the position of the rotation pole with respect to the Earth's crust and mantle to within about 1 milli-arcsec [e.g., Mueller and Zerbini, 1989; IERS Annual Report for 1988].

Change in the length-of-day is typically determined from UT1 by some numerical differentiation scheme thereby introducing further uncertainty in the determination of the length-of-day. The mean position of the rotation pole (ψ) can be recovered from the location of the instantaneous rotation pole (i.e., from $m_2$ and $m_0$) by either numerical differentiation or by deconvolution techniques [e.g., Gross and Chao, 1985] thereby introducing further uncertainty in its determination. Thus the coseismic effect of the Macquarie Ridge earthquake upon the length-of-day is a factor at least 2000 times smaller than current observational accuracies of the length-of-day, and the amplitude of the change in the mean position of the rotation pole is at least 10 times smaller than can be currently detected.

Not withstanding the small size of these predicted effects of the Macquarie Ridge earthquake, observations of the Earth's rotational properties were examined for step-like changes in the length-of-day and ψ that may have been caused by this event. All available independent, high-quality space-geodetic observations of UT1 and polar motion were combined [Gross et al., 1989] in a rigorously self-consistent manner by a technique based upon the Kalman filter [Morabito et al., 1988]. In addition to producing smoothed, interpolated estimates of UT1 and polar motion, the Kalman filter also estimates the length-of-day and location of the mean rotation pole. A year-long series of these length-of-day and ψ estimates, having formal uncertainties about 0.03 ms and 15 mas, respectively, centered on the time of the Macquarie Ridge earthquake was examined, but no signal was found that could be ascribed to it, either in the length-of-day, or in ψ.

Even though the effect of the Macquarie Ridge earthquake on the Earth's rotational properties is too small to be presently detected, signals of this size may soon be detectable. Observational systems are being continually improved, leading to ever smaller uncertainties in the observations of Earth rotation and polar motion. In fact, it is currently being proposed [e.g., Mueller and Zerbini, 1989; Dickey and Shutz, 1989] that these improvements be such as to yield accuracies of about 0.1 milli-arcsec in the determination of polar motion, and of about 6 µsec in the determination of UT1. At this proposed level of accuracy, the effect of the Macquarie Ridge earthquake on the length-of-day would still not be observable, but its effect upon polar motion might be.

Table 1 also gives the expected coseismic effects on the Earth's rotational properties of the 1960 Chile, 1964 Alaska, and 1977 Sumba earthquakes. It is seen that the 1960 Chilean earthquake, whose moment release was some 200 times greater than that of the Macquarie Ridge event, is estimated to have shifted the mean position of the rotation pole by 23 milli-arcsec. A present-day change this large could certainly be observed given current observational accuracies. The 1960 Chilean earthquake is also estimated to have decreased the length-of-day by 8.4 µsec, too small to be presently detected but perhaps detectable at the proposed future level of accuracy.

Table 2 presents the estimated coseismic effect of the Macquarie Ridge earthquake upon selected low-degree Stokes coefficients. It is found that these coefficients should have changed by about 1 part in $10^3$. The changes $AC_{22}$ and $DS_{22}$ have been discussed above in regard to the mean position of the rotation pole (i.e., the polar motion excitation ψ). The changes $AC_{22}$ and $DS_{22}$ can be interpreted [e.g., Chao and Gross, 1987] as increasing the difference between the two equatorial principal moments of inertia by $4.4 x 10^{-12} M a^2$ and changing their direction by 6.0 milli-arcsec towards the West in the plane of the equator.

Recent static gravitational field models derived solely from satellite tracking data are able to determine the low-degree Stokes coefficients to a precision no better than about $10^{-10}$ [e.g., Marsh et al., 1988]. However, Yoder et al. [1983], Rubincam [1984], and Cheng et al. [1989] have reported the detection of temporal changes in low-degree zonal Stokes coefficients on the order of a few parts in $10^{-12}$ yr$-1$ [note that their $J_2 = -(2\pi^2/\Omega)^2 C_{20}$]. Furthermore, accuracies of about one part in $10^{-9}$ or better in the determination of static Stokes coefficients can perhaps be achieved in the future either by a satellite capable of observing the Earth's gravitational field such as a gravity gradiometer mission which would especially

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<th>TABLE 1. The Modeled Coseismic Effect of Earthquakes on the Earth's Rotation Vector. (Units of $M_o$ are $10^{21}$ Nm)</th>
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improve the determination of the high-degree coefficients, or by placing GPS (Global Positioning System) receivers onboard orbiting spacecraft which would especially improve the determination of the low-degree coefficients [e.g., Paik et al., 1988; Colombo, 1988; Smith et al., 1988; Pavlis and Smith, 1989]. Somewhat higher accuracies can in principle be achieved for detecting temporal changes, especially in the low-degree zonal coefficients [Colombo, personal communication, 1990].

Thus changes in the low-degree Stokes coefficients caused by the 1977 Sumba or 1989 Macquarie Ridge earthquakes could perhaps be detected in the future by improved geodetic measuring techniques. The changes caused by the 1960 Chile or 1964 Alaska events could certainly be detected by such techniques; in fact, they could probably be detected by currently available techniques. For example, the 1964 Alaska event is estimated (Table 2) to have instantly changed $C_{20}$ by an amount that would take post-glacial rebound 2 years to undo according to current observations of the secular rate of $C_{20}$ change [Yoder et al., 1983; Rubincam, 1984; Cheng et al., 1989].

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