Variations of Mars Gravitational Field and Rotation
Due to Seasonal CO2 Exchange

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About a quarter of the Martian atmospheric mass is exchanged between the atmosphere and the polar caps in the course of a Martian year: CO2 condenses to form (or add to) the polar caps in winter and sublimes into the atmosphere in summer. This paper studies the effect of this CO2 mass redistribution on Martian rotation and gravitational field. Two mechanisms are examined: (1) the waxing and waning of solid CO2 in the polar caps and (2) the geographical distribution of gaseous CO2 in the atmosphere. In particular, the net peak-to-peak changes in \( J_2 \) and \( J_3 \) over a Martian year are both found to be as much as \(-6 \times 10^{-9}\). A simulation suggests that these changes may be detected by the upcoming Mars Observer under favorable but realistic conditions.

1. INTRODUCTION

Mass redistribution in or on a planet will change the planet’s gravitational field. It will also change the planet’s rotation via the conservation of angular momentum. The present paper studies these effects caused by the seasonal redistribution of the carbon dioxide (CO2) mass between the Martian atmosphere and the polar caps.

It is well known that Mars has seasons like the Earth. Yet Mars’ colder atmospheric temperature (in average \(-70^\circ\text{K}\) lower than the Earth’s) and its predominantly CO2 atmosphere (more than 95% concentration) conspire to create a large exchange of CO2 mass between the atmosphere and the polar caps. This exchange occurs in seasonal cycles. Every winter a sizable fraction of the CO2 atmosphere freezes out at the appropriate pole to make a larger polar cap. In summer the cap sublimes, returning CO2 gas to the atmosphere. The CO2 mass involved is estimated to be at least \(8.1 \times 10^{15}\) kg [Hess et al., 1980], which amounts to some 25% of the total Martian atmosphere, in contrast to the Earth’s atmosphere which only varies by \(-0.02\%\) primarily because of seasonal variations in the water vapor content.

Qualitatively, global-scale gravitational and rotational effect is proportional to the ratio of the net redistributed mass to the planet’s total mass. For the Martian CO2 exchange this ratio is about \(1.3 \times 10^{-6}\). No terrestrial phenomenon is known to produce a relative mass redistribution of such magnitude in a year. For example, the postglacial rebound of the Earth’s mantle, the strongest El Niño events in the atmosphere-ocean system, and the greatest earthquakes are all smaller by at least 1–2 orders of magnitude [e.g., Rubincam, 1984; Gross and Chao, 1985; Chao and Gross, 1987].

In this paper, two concurrent mechanisms will be examined: the polar condensation/sublimation of solid CO2 and the global distribution of gaseous CO2 over the topography. In addition, we shall present simulation results which indicate the possibility of detecting these effects geodetically by the upcoming Mars Observer mission. Some aspects of the rotational effect of the CO2 mass redistribution have been previously discussed by Reasenberg and King [1979] and Cazenave and Balmino [1981] (but see section 3). The possibility that the redistribution has increased Mars obliquity by \(1^\circ\) or \(2^\circ\) over the age of the solar system has been studied by D. P. Rubincam (Mars secular obliquity change due to the polar ice caps, submitted to Celestial Mechanics, 1990).

2. GENERAL FORMULATION

Let the origin of the coordinate system be at Mars’ center of mass, and let the \(x\), \(y\), and \(z\) axes point to the \(0^\circ\) Meridian, the \(90^\circ\) Meridian, and the North Pole, respectively. The external gravitational potential \(U\) can be expressed as [e.g., Kaula, 1966]

\[
U(r') = \frac{GM}{r'} \left\{ 1 + \sum_{l=2}^{\infty} \frac{1}{l} \sum_{m=0}^{l} \frac{1}{(l-m)!} \left( \frac{R}{r'} \right)^{l-1} C_{lm}(r') \right\}
\]

where \(G\) is the gravitational constant, \(M\) is the total mass and \(R\) the mean radius of the planet, \(\Omega\) is the solid angle representing the colatitude \(\theta\) and the longitude \(\lambda\), and \((r', \theta', \lambda') = (r', \theta, \lambda)\) locates the field point \(r'\). \(Y_{lm}\) is the spherical harmonic function of degree \(l\) and order \(m\), with the index \(i = 1, 2\), designating, respectively, the cosine or sine term in \(\lambda\), normalized according to

\[
\int Y_{lm}(\Omega) Y_{l'm'}(\Omega) \, d\Omega = 4\pi \delta_{ll'} \delta_{mm'}
\]

The triply indexed coefficients \(C_{lm}\) are related to the usual Stokes coefficients and the zonal \(J\) coefficients by

\[
C_{lm1} = C_{lm}
\]

\[
C_{lm2} = S_{lm}
\]

\[
J_l = -(2l + 1)^{1/2} C_{l0}
\]

They can be shown [e.g., Chao and Gross, 1987] to be normalized multipoles of the density \(\rho(r)\):

\[
C_{lm} = \frac{1}{(2l + 1)MR^l} \int V \rho(r)r^l Y_{lm}(\Omega) \, dV
\]

where the integration is over the volume \(V\) of the planet (including the atmosphere).
Now suppose the mass distribution has a (small) time-varying component, which gives rise to a (small) variation in $C_{imi}$ according to (4). Suppose further that this mass variation occurs in a thin veneer on the planet surface (within the atmosphere for the Martian CO$_2$ exchange), so that a good approximation we can take $r = R$ in (4) to evaluate this effect. In a Eulerian description, (4) then reduces to an integral over the unit sphere:

$$
\Delta C_{imi}(t) = \frac{R^2}{(2I + 1)M} \int \Delta \sigma(\Omega, t) Y_{imi}(\Omega) \, d\Omega \tag{5}
$$

where $\Delta \sigma(\Omega, t)$ is the surface density of the mass variation.

The quantity $\Delta \sigma$ actually consists of two parts: the "apparent" surface mass change as if solid Mars were rigid and the elastic yielding of solid Mars in response to surface loading and unloading. In a linear rheology the latter is proportional to the former, through the factor of the load

$$
\text{where } A_{\text{tr}}(\Omega, t) = \text{surface density of the mass variation.}
$$

The excitation of the polar motion, usually expressed in Cartesian components, $\Psi(t) = \Psi_x(t) + i \Psi_y(t)$, gives the angular shift in the figure axis due to mass redistribution. For any mass redistribution that occurs on the surface of the planet and subject to the conservation of mass, the LOD variation is proportional to $\Delta J_2$ [e.g., Chao et al., 1987]:

$$
\Delta \text{LOD}(t) = \text{LOD} \left[ \frac{2MR^2 \Delta J_2(t)}{3C} \right] \tag{6}
$$

where $C$ is the greatest moment of inertial of the planet. For Mars, LOD = 88,775 s, and we adopt a value of 0.345 for $\text{ClMR}^2$ [Bills, 1989] (but also see Kaula et al., [1989]), so $\Delta \text{LOD}(t) = 1.72 \times 10^8 \text{J}_2(t)$ in milliseconds (ms).

The CO$_2$ mass variation is estimated from the atmospheric pressure recordings of the two Viking landers [Hess et al., 1980; Tillman, 1985]. It is shown schematically in Figure 1, where the subscripts $n$ and $s$ indicate the north and south caps, respectively, $P_I$ is the Legendre function of degree $I$, and $\Delta M_{n,s}(t) = 2\pi R^2(1 - \cos \nu) \Delta \sigma(\Omega, t)$ is the total CO$_2$ mass accumulation on the cap at any given moment.

$$
\Delta \text{LOD}(t) = \frac{1}{(2I + 1)^{1/2}(1 - \cos \nu)} \int_{-\pi}^{\pi} P_I(\mu) \, d\mu \left[ \frac{\Delta M(t)}{M} \right]_n
$$

accompany each other inasmuch as the total CO$_2$ mass is conserved. The total effect is simply the sum of the two contributions.

3.1. Waxing and Waning of Polar Caps

First we assume a simple geometry where the caps are circular and centered at the poles, extending to a constant angle $\kappa$ from the poles. Let the seasonal changes occur uniformly, i.e., $\Delta \sigma(\Omega, t) = \Delta \sigma(t)$, on the polar caps (see later text for more realistic geometries).

For $m \neq 0$, (5) gives $\Delta C_{imi} = 0$. This is a direct consequence of the axial symmetry of the assumed geometry. For $m = 0$, (5) leads to

$$
\Delta \sigma(t) = \int_{-\pi}^{\pi} P_I(\mu) \, d\mu \left[ \frac{\Delta M(t)}{M} \right]_n + (-1)^{I}(\text{same}) \tag{8}
$$

where the subscripts $n$ and $s$ indicate the north and south caps, respectively, $P_I$ is the Legendre function of degree $I$, and $\Delta M_{n,s}(t) = 2\pi R^2(1 - \cos \nu) \Delta \sigma(t)$ is the total CO$_2$ mass accumulation on the cap at any given moment.

The CO$_2$ mass variation is estimated from the atmospheric pressure recordings of the two Viking landers [Hess et al., 1980; Tillman, 1985]. It is shown schematically in Figure 1, which gives the sum of $\Delta M_{n}(t)$ and $\Delta M_{s}(t)$. The zero level corresponds to the minimum amount of CO$_2$ in the polar caps which occurs around the northern winter solstice (areocentric longitude of the Sun $L_s = 270^\circ$), designated by $t = 0$. The time of maximum amount of CO$_2$ in polar caps is designated $t_0$.

The angular extent $\kappa$ of the CO$_2$ polar caps can be estimated from Viking Orbiter imagery data [Briggs et al., 1985].
Balmino [1981] obtained ALOD values that are smaller by Borderies et al., 1980.

predicted position of 1 year later based on the LOD at t = 0 magnitude larger than its terrestrial counterpart [Cheng et al., 1989; Au and Chao, 1990]. Whether they can be detected about 30 mm in the course of 1 Martian year.

Amplitude (8) becomes progressively smaller with increasing I. In the following discussion we shall only consider a few lowest-degree terms by evaluating their maximum amplitudes which occur at the peak of formation of the south cap in late northern summer, indicated by t = t0 in Figure 1. Thus substituting into (8) Aml(t0) = 8.1 x 10^15 kg [Hess et al., 1980] and \( \kappa_s = 45^\circ \), we get \( \Delta C_{B}(t_0) \) and the corresponding \( \Delta J_l(t_0) \). They are presented in Table 1, where the zero level is the same as in Figure 1.

For even I, \( |\Delta C_{B}(t_0)| \) is also the annual peak-to-peak variation. For odd I the annual peak-to-peak variation is somewhat larger than \( |\Delta C_{B}(t_0)| \). This is because of the addition of the contributions from the north cap, which has the opposite sign (see (8)) and the opposite seasonality as the south cap. The amplification is about 50% judging from the relative level of the north cap in Figure 1.

The maximum change in J1, \( \Delta J_1(t_0) = 10.6 \times 10^{-9} \), corresponds to 21 mm of a shift of the center of mass of the solid Mars along the z axis, or a peak-to-peak magnitude of about 30 mm in the course of 1 Martian year.

The maximum change in J2, \( \Delta J_2(t_0) = -7.5 \times 10^{-9} \), is significant. In comparison, it is some 50 times larger than the observed seasonal \( \Delta J_2 \) for the Earth caused by climate [Cheng et al., 1989; Au and Chao, 1990]. The change in J3 is also considerable: \( \Delta J_3(t_0) = 4.0 \times 10^{-9} \), again an order of magnitude larger than its terrestrial counterpart [Cheng et al., 1989; Au and Chao, 1990]. Whether they can be detected by the Mars Observer will be discussed in association with orbit simulations in section 4.

Using (6), the maximum change in LOD is found to be \( \Delta \text{LOD}(t_0) = -1.30 \text{ ms} \), which is comparable in amplitude to the seasonal \( \Delta \text{LOD} \) on Earth. Integrating this shortage in LOD (relative to the zero level of Figure 1, of course) over one entire (Martian) year, one gets an equatorial surface displacement of roughly 100 m. Thus, for instance, if one marks a surface point on the Martian Equator, then the predicted position of 1 year later based on the LOD at t = 0 (which is the longest day in 1 year) would lag ~100 m behind the true position. Whether this displacement is discernible from existing tracking data for the Viking landers remains an interesting proposition [cf. Reasenberg and King, 1979; Borderies et al., 1980].

Both Reasenberg and King [1979] and Cazenave and Balmino [1981] obtained \( \Delta \text{LOD} \) values that are smaller by more than a factor of 2. Reasenberg and King did not give any details of their calculation. The formulation of Cazenave and Balmino, on the other hand, fails to satisfy the conservation of mass: the polar caps were treated as extra mass, and the corresponding (uniform) decrease in the atmospheric mass (and hence its moments of inertia) was not taken into account.

In principle, the angular momentum change associated with the zonal winds (accompanying the CO2 mass redistribution) can also change LOD. This contribution, however, is found to be insignificant [Cazenave and Balmino, 1981], in contrast with the Earth where the zonal wind is the dominant source for seasonal LOD changes.

The values in Table 1 are calculated for a uniform condensation/sublimation of solid CO2 over the polar caps. The true distribution is probably more concentrated toward the poles. This will result in moderately larger changes; Table 1 represents a lower bound in this sense. For the extreme case where all the condensed CO2 is concentrated at the pole gives a value of \( \Delta J_2(t_0) = -12.5 \times 10^{-9} \), whereas a "layered cake" model where the CO2 layer between \( \kappa = 0^\circ \) and 22.5\(^\circ\) is twice as thick as that between \( \kappa = 22.5^\circ \) and \( \kappa = 45^\circ \) gives \( \Delta J_2(t_0) = -8.7 \times 10^{-9} \).

The CO2 polar caps may not be axially symmetric as assumed. There usually exists a (variable) offset (call it \( \delta \)) between the pole and the center of mass of the polar cap, as clearly exhibited by the south cap during regression phase [e.g., Iwasaki et al., 1989]. For \( \delta \ll R \) this introduces an additional contribution to \( \Delta C_{B} \) that is proportional to \( \delta/R \). For \( C_{B} \) coefficients this error is relatively small and negligible. However, it results in nonzero values for \( m \neq 0 \) terms (which are zero otherwise). In particular, it can be shown (equation (7)) that an angular offset of, say, 1\(^\circ\) will produce a polar motion excitation of about 20 milliarcseconds (mas) in magnitude. At present, however, observational data are insufficient to constrain \( \delta \) realistically.

### Table 1. Maximum Gravitational and Rotational Effects of the Changing Polar Cap CO2 That Occurs at \( t = t_0 \)

<table>
<thead>
<tr>
<th>I</th>
<th>( \Delta C_{B} \times 10^{-9} )</th>
<th>( \Delta J_1 \times 10^{-9} )</th>
<th>Rotational Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-6.1</td>
<td>10.6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.4</td>
<td>-7.5</td>
<td>( \Delta \text{LOD} = -1.30 \text{ ms} )</td>
</tr>
<tr>
<td>3</td>
<td>-1.5</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
<td>-0.94</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
<td>-1.0</td>
<td></td>
</tr>
</tbody>
</table>

The zero reference level is as in Figure 1.

3.2. Geographical Distribution of the Atmosphere

To conserve mass in the seasonal CO2 exchange, any mass change in the polar caps is accompanied by the reverse change in the atmosphere (cf. Figure 1). The above would be the total effect if the changing CO2 mass distribution in the atmosphere is uniform geographically. In reality, the Martian surface topography will influence this distribution, making it nonuniform. That, in turn, gives rise to additional effects in the gravitational field and rotation. How large are these effects?

For simplicity we assume that the atmosphere is always in hydrostatic equilibrium. Physically, this requires an instantaneous redistribution of CO2 mass over the globe. It is permissible since we are only concerned with seasonal periods which are much longer than the time scale for Martian atmospheric circulation.

In equilibrium, any isopycnic (constant density) surface of the atmosphere will follow an equipotential surface of the gravitational field. In a hypothetical case where the surface topography is itself a geoid (for example, if the entire planet were covered by an ocean), the geographic distribution of...
the atmosphere will be practically uniform. Its surface density \( \rho(\Omega, t) \) will be independent of geographic location \( \Omega \), changing only with time \( t \); and the changing atmosphere has no gravitational effect (except on the total mass which, of course, is compensated by the changing polar caps).

In reality, the Martian surface topography is far from being a geoid; Mars is noted for its large surface relief, so much so that a substantial fraction of the atmosphere can "feel" the topography. The atmospheric surface density \( \rho \) will be smaller where the topography protrudes the geoid (e.g., over a high plateau) and will be larger otherwise (e.g., over a low basin).

Thus it is the departure of the true topography from the geoid that determines how \( \rho(\Omega) \) deviates from being uniform. Similar to (1), this "effective topography" (Figure 2) can be written as

\[
h(\Omega) = R \left[ \sum_{l=1}^{\infty} \sum_{m=0}^{l} \sum_{i=1}^{2} H_{lmi} Y_{lmi}(\Omega) \right]
\]

(9)

The coefficient \( H_{lmi} \), using (1) and Bruns formula [e.g., Heiskanen and Moritz, 1967], can be obtained by

\[
H_{lmi} = H_{lmi} - C_{lmi}
\]

(10)

where \( H_{lmi} \) is the harmonic coefficient relating to the true topography \( h'(\Omega) \) through a relationship similar to (9). The gravitational \( C_{lmi} \) estimates have been obtained by Christensen and Balmino [1979] and Balmino et al. [1982] up to degree and order 18. The topographic \( H_{lmi} \) series has been given to degree and order 16 by Bills and Ferrari [1978] on the basis of occultations and radar, spectral, and photogrammetric measurements, although uncertainties as large as 1 km in topography are not uncommon [Lindal et al., 1979]. Note that the values for \( i = 2 \) coefficients in these studies should change sign to conform to our present (east) longitude \( \lambda \). Here we include the first degree harmonics, which represent the displacement of the center of the figure from the center of mass.

From \( h(\Omega) \) one can obtain \( \Delta \rho(\Omega, t) \) as follows. The lower atmosphere is where virtually all the atmospheric mass resides. The equilibrium density variation there can be approximated by the isothermal profile

\[
\rho(z, \Omega) = \rho_0 \exp \{ -[z + h(\Omega)]/Z \}
\]

(11)

where \( z \) is the vertical height from the surface, and \( \rho_0 \) is the atmospheric density on the geoid \( z = -h(\Omega) \). Z is the atmospheric scale height (Figure 2) and, for an ideal gas, is given by \( Z = kT/\rho g \), where \( k = 1.38 \times 10^{-23} \text{ J K}^{-1} \) is the Boltzmann constant, \( T \) is the temperature of the lower atmosphere, \( \nu = 7.3 \times 10^{26} \text{ kg} \) is the molecular weight of \( \text{CO}_2 \), and \( g = 3.7 \text{ m s}^{-2} \) is the surface gravity.

Thus the surface density variation of the atmospheric \( \text{CO}_2 \) is

\[
\Delta \rho(\Omega, t) = \int_0^\infty \Delta \rho(z, \Omega, t) \, dz = \Delta \rho_0(t)Z \exp \{ -h(\Omega)/Z \}
\]

(12)

To proceed, we notice that although the true Martian topography \( h' \) is at places comparable to (or even exceeds) \( Z \), the effective topography \( h \) is generally much smaller. This is a consequence of the high correlation between Martian topography and gravitational field [e.g., Bills and Ferrari, 1978], especially in the low-degree harmonics on which we shall concentrate. Therefore we can approximate \( \exp \{ -h(\Omega)/Z \} \) by substituting the expression (9) for \( h(\Omega) \) and retaining the first-order terms in a Taylor expansion:

\[
\exp \{ -h(\Omega)/Z \} = 1 - \frac{R}{Z} \left[ \sum_{l=1}^{\infty} \sum_{m=0}^{l} \sum_{i=1}^{2} H_{lmi} Y_{lmi}(\Omega) \right]
\]

(13)

Substituting this expression into (12) and then (5) and using the orthogonality relation (2), we finally obtain

\[
\Delta C_{lmi}(t) = -\frac{1}{2l+1} \frac{RH_{lmi} \Delta M(t)}{Z M}
\]

(14)

where \( \Delta M(t) = 4\pi R^2 Z \Delta \rho_0(t) \) is the total \( \text{CO}_2 \) mass added to the atmosphere; \( \Delta M(t) = -[\Delta M_s(t) + \Delta M_r(t)] \), as shown in Figure 1.

Table 2 presents the maximum amplitude of \( \Delta C_{lmi}(t) \), which occurs at \( t = t_0 \), for some most prominent low-degree harmonics. Again, the zero level is the same as in Figure 1.

The first degree \((l = 1)\) harmonics gives the change in the relative displacement of the center of the figure from the center of mass [Bills and Ferrari, 1978]. This displacement is
found to be a rather small 2.4 mm in the direction (92°E, 44°N).

The maximum change in J2, ΔJ2(t0) = 1.6 × 10^{-9}, is again the largest term, reflecting the large deviation of solid Mars from the state of hydrostatic equilibrium. It is out of phase with the polar cap contribution, decreasing the amplitude by about one fifth. These two changes in J2 are out of phase because of Mars' excess topographic oblateness. The polar regions are topographically low. Hence the excess atmosphere there decreases when the caps grow, increasing J2. The maximum change in LOD is immediately given by (6): ΔLOD(t0) = 0.28 ms, about one fifth of the polar cap effect but of opposite sign.

The excitation function Ψ(t) for the polar motion due to the atmospheric CO2 change can be obtained from (7) and Table 2. It stays strictly along a fixed meridian, namely 125°E, simply because the topography is fixed. Its maximum amplitude is 13 milliarcsecond (mas), equivalent to a surface polar shift of about 21 cm. This is a fraction of the annual wobble excitation on Earth produced by meteorological and hydrological variations [Chao and Au, 1989]. Note that this excitation, at annual (669 Martian days) and semiannual periods, is not particularly efficient to drive the polar motion as in the case of the Earth. This is because Mars, assuming an Earth-like rigidity, has a free wobble period of about 200 Martian days and hence no near resonance should be expected from seasonal excitations.

The change in J3 is also out of phase with respect to the polar cap contribution, but its amplitude is negligible. The (l = 2, m = 2) and (l = 3, m = 3) harmonics are the only other components that produce considerable changes for Mars, owing to the existence of the extensive Tharsis construct near the Equator and its antipodal, but smaller, uplift.

4. Discussion

Combining Tables 1 and 2 gives the net effect of the seasonal CO2 exchange on Martian gravitational field and rotation. The most prominent changes are in J2 and J3. The estimated annual peak-to-peak change in J2 is ~6 × 10^{-9}. For J3 this change (obtained by multiplying the maximum amplitude by a nominal factor of 1.5; see section 3.1) is also ~6 × 10^{-9}. Since these are probably underestimated (section 3.1), the rule of thumb is that Martian J2 and J3 change by 10^{-8} annually.

It is interesting to investigate whether the upcoming Mars observer (MO) mission can detect these changes. In a series of simulations, Smith et al. [this issue] obtain separate geopotential solutions for three selected 60-day sessions during the mission. The results indicate that MO can determine J2 and J3 to an accuracy typically of the order of 10^{-8}, depending on various factors.

As a specific example, let us examine the J2 and J3 determination based on one scenario that we consider to be representative. For random observation noise we assume a single-station Doppler radio tracking with a precision of 0.3 mm s^{-1} in the line-of-sight speed integrated over 1-min intervals. For systematic modeling biases, three nongravitational forces that act on the spacecraft are taken into consideration: the Martian atmospheric drag, the solar radiation pressure, and the spacecraft's angular momentum desaturation residue accelerations (AMDRA) that are necessary for orbit corrections and maneuvers. A 15% error in the modeling of the atmospheric drag is adopted, whereas the radiation pressure is solved for. The AMDRA, each one imparting a change of 10^{-7} m s^{-1} in speed, are assumed to occur once every two revolutions and are completely unmodeled. These assumptions tend to be conservative in the sense that the actual observation scheme, data quality, and force models may be more favorable [Smith et al., this issue]. The resultant root-mean-square errors are found to range from 0.65 to 2.4 × 10^{-8}, with errors in J2 somewhat smaller than errors in J3.

The fact that these errors are of the same order of magnitude as the predicted signal amplitude of J2 and J3 variations implies the possibility of detection of the latter in the following way. Conceivably, one can obtain a consecutive time series of N solutions (e.g., one for every 30 days) over the course of a Martian year (the nominal lifetime of the MO mission) and fit seasonal sinusoids to the time series. The effective noise level then is reduced by a factor of N^{1/2}. Alternatively, one can incorporate the sinusoids in the solution simultaneously with the static geopotential. This presumably can reduce the residuals, and the detectability hinges upon whether the actual reduction is statistically significant. The orbit determination program GEODYN has been modified to accommodate this requirement.

There are other possibilities to improve further the determination of the Martian gravitational field. A better modeling of the Martian atmospheric drag via knowledge gained from the MO mission can significantly reduce the associated estimation biases. Surface altimetry data as differences at orbit crossovers, which presumably will be available from the MO laser altimeter, can provide further information about the MO orbit. Finally, the raised MO orbit recommended by Smith et al. [this issue] can result in a much improved determination of the geopotential via a lower atmospheric drag and the alleviation of the large effects seen in the error spectrum of the spacecraft resonant orders. The latter is particularly significant for the low-degree harmonics (F. Lerch, private communication, 1989).

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References


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