Snow Load Effect on the Earth's Rotation and Gravitational Field, 1979–1985

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A global, monthly snow depth data set has been generated from the Nimbus 7 satellite observations using passive microwave remote-sensing techniques. In this paper we analyze 7 years of data, 1979–1985, to compute the snow load effects on the earth's rotation and low-degree zonal gravitational field. A uniform sea level decrease has been assumed in order to conserve water mass. The resultant time series show dominant seasonal cycles. The annual peak-to-peak variation in \( J_2 \) is found to be \( 2.3 \times 10^{-19} \), that in \( J_4 \) to be \( 1.1 \times 10^{-19} \), and believed to decrease rapidly for higher degrees. The corresponding change in the length of day is \( 41 \mu \) s. The annual wobble excitation is \( (4.9 \text{ marc sec, } -109^\circ) \) for the prograde motion component and \( (4.8 \text{ marc sec, } -28^\circ) \) for the retrograde motion component. The excitation power of the Chandler wobble due to the snow load is estimated to be about 25 dB less than the power needed to maintain the observed Chandler wobble. The superior quality of the satellite data over conventional data acquired by ground observations and modeling is demonstrated. We also discuss the role of atmospheric water and the problems arising from the lack of snow load observations over the Antarctic and Greenland ice sheets.

I. INTRODUCTION

Any change in mass distribution inside or on the earth results in two global geodetic effects: (1) It changes the earth's gravitational field according to Newton's gravitational law, and (2) it changes the earth's rotation according to the principle of conservation of angular momentum, producing changes in polar motion and length of day. Examples include tides, earthquakes, meteorological variations, mantle movement associated with postglacial rebound and plate tectonics, and core activities. The geodetic effect of one meteorological variation, namely, continental snow, is the subject of the present study.

Snow is one important, and perhaps the most visible, component of the continental water storage. The latter also includes (1) soil moisture stored in the plant root zone (and available for evapotranspiration), (2) underground water storage percolated down below the root zone to the groundwater table, and (3) surface water and river runoff that has not found its way back to the ocean. The excitation of the earth's annual wobble due to the seasonal mass redistribution of air and water has been under study since the turn of the century (for reviews, see Munk and MacDonald [1960] and Lambeck [1980]). The annual wobble is a forced motion of the earth's rotation axis; it, together with the 14-month Chandler wobble, compose the periodic polar motion. The greatest contribution to the excitation of the annual wobble was determined early to be of atmospheric origin. A lesser but significant contribution comes from the water storage variations on continents, and it is this excitation that is now the least accurately determined for the annual wobble [e.g., Wahr, 1983].

The first, rudimentary calculation of continental water storage excitation of the annual wobble was made by Jeffreys [1916], who observed that there would be contributions from both global snow load and soil moisture. These contributions have different seasonal and geographical distributions. He neglected soil moisture without justification and calculated the excitation based on rough temporal and spatial estimates of snow depth over Eurasia and North America. Jeffreys also considered the seasonal mass load due to vegetation. These values were later quoted by Jeffreys [1976].

Systematic methods of calculating continental water storage using regional water budgets started with the work of Thornthwaite [1948]. A subsequent study by Van Hylckama [1956] obtained monthly values of the water storage for 10° by 10° squares of longitude and latitude. Climatological and hydrological researches have recently been extended to study further the global water storage distributions. A global water budget by Willmott et al. [1985] includes snow storage, which is related to precipitation and air temperature observations. A global soil moisture sensitivity study has been reported by Minz [1984], and evapotranspiration fluxes are now being used as one of the boundary conditions in general circulation models [Kalnay et al., 1983].

Van Hylckama's [1956] data were used by Munk and MacDonald [1960] to calculate the continental water storage excitation of the annual wobble. The global water budget and subsequent wobble excitation calculations were later revised by Van Hylckama [1970]. Recent studies by Hinnov and Wilson [1987] and Close and Stolz [1987] have made revised calculations using different data sets and model parameterization. These studies yielded substantially different results, probably because of the inherent shortcomings and limitations in conventional data sets. In this paper we shall emphasize and demonstrate the importance of more accurate data, such as provided by satellite observations, in estimating polar motion excitations.

Unlike polar motion, the seasonal change in length of day (LOD) is primarily due to atmospheric motion [e.g., Rosen and Salstein, 1983]; the contribution from the atmospheric mass redistribution is relatively small. The effects on LOD due to surface mass redistribution associated with other meteorological processes are presumably even smaller, and little effort...
has been given to its study. The same is true for studies on the
earth’s gravitational field changes due to surface mass redistri-
butions. In fact, not until recently could the changes in the
gravitational field, particularly those in low-degree zonal
harmonics, be detected from geodetic satellite orbit determi-
nations [Yoder et al., 1983; Alexander, 1983; Rubincam, 1984].

The present paper studies the variations in the earth’s gravi-
tational field and rotation (polar motion and LOD) due to the
combined effect of the seasonal loading and unloading of
snow on continents and the corresponding variation in the
amount of water in the ocean. Section 2 outlines the develop-
ment of the formulae used in the calculation. These for-
mulae have been used by Chao and Rubincam [1986] in a study
of the geodetic effects of the seasonal CO$_2$ mass shift on mars,
and by Chao and Gross [1987], in a different form, for the
geodetic effects of mass shift induced by earthquakes. Section
3 describes the snow depth data set that has recently been
derived from satellite observations for the period 1979-1985.
The results are then presented and discussed in sections 4 and
5. An earlier version of the data set has been used by Chao et
al. [1987] to compute the annual wobble excitation for the

2. FORMULATION

Let the coordinate system be such that the origin is at the
earth’s center of mass, and the $x$, $y$, and $z$ axes point to the $0^\circ$E
(Greenwich) meridian, the $90^\circ$E meridian, and the north pole,
respectively. The anomalous gravitational potential $U$ (with
respect to the reference spheroid) for the exterior of the earth
satisfies Laplace’s equation and can be expressed through the
spherical harmonic expansion in the following form:

$$U(r_0) = \frac{GM}{r_0} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \frac{R}{r_0^l} A_{lm} Y_{lm}^{*}(\Omega_0)$$

where $R$ denotes the real part of a complete quantity, $\Omega$ is an
abbreviation for the colatitude $\theta$ and east longitude $\lambda$, $(r_0, \theta_0, \lambda_0)$ is the spherical coordinate of the field point $r_0$,
$G$ is the gravitational constant, $M$ is the total mass, and $R$ is the
mean radius of the earth. The spherical harmonics are defined by

$$Y_{lm}^{*}(\Omega) = [(2 - \delta_{ml})(2l + 1)(l - m)!/(l + m)!]^{1/2} \cdot P_{lm}(\cos \theta) \exp (im\lambda)$$

where $\delta$ is the Kronecker delta function, $P_{lm}$ is the associated
Legendre function of degree $l$ and order $m$. They are normal-
ized according to

$$\int Y_{lm}(\Omega) Y_{lm}^{*}(\Omega) d\Omega = 4\pi(2 - \delta_{ml}) \delta_{ll'} \delta_{mm'}$$

where the asterisk denotes complex conjugation, $d\Omega = \sin \theta d\theta
d\lambda$ is an element of solid angle, and the integration is over the
unit sphere. The normalization is chosen such that the harmonic
coefficient $A_{lm}$ in (1), here referred to as the complex
Stokes coefficient, is directly related to the ordinary (normal-
ized) Stokes coefficients $C_{lm}$ and $S_{lm}$ [cf. Kaula, 1966] in a
simple manner:

$$A_{lm} = C_{lm} + iS_{lm}$$

By comparing equation (1) with the multipole expansion of $U$
[e.g., Jackson, 1966] one can relate the complex Stokes
coefficients to the density distribution $\rho(t)$ of the earth [e.g.,
Chao and Gross, 1987]:

$$A_{lm} = \frac{1}{(2l + 1)M R^l} \int \rho(r)r^l Y_{lm}^{*}(\Omega) dV$$

where $dV$ is the volume element and the integration is over the
entire volume $V$ of the earth.

Now suppose a time-varying density change $\Delta \rho(r, t)$ is im-
posed upon $\rho(r)$, and we want to find the resultant change in
$A_{lm}$. There are two approaches to formulate this problem: the
Lagrangian approach that follows individual mass particles and
the Eulerian approach that focuses on individual points in
space. In the present application, the latter is the natural
choice. In an Eulerian description, equation (5) immediately
gives

$$\Delta A_{lm}(t) = \frac{1}{(2l + 1)M R^l} \int \Delta \rho(r, t)r^l Y_{lm}^{*}(\Omega) dV$$

where the integration is over the entire surface of the earth.

For global scale meteorological processes that occur in a
thin veneer on the earth’s surface, such as the seasonal snow
and water mass redistribution, any radial movement is negligi-
ble compared to lateral movements. Therefore, to a good ap-
proximation we can take $r = R$ in equation (6) and use surface
density $\Delta \rho(\Omega, t)$ in place of the volume density $\Delta \rho(r, t)$. Equation (6) then reduces to

$$\Delta A_{lm}(t) = \frac{1}{(2l + 1)M} \int \Delta \rho(\Omega, t) Y_{lm}^{*}(\Omega) d\Omega$$

where the integration is over the entire surface of the earth.

Following Munk and MacDonald [1960], it is recognized that $\Delta \rho$
consists of two parts: the surface density change $\Delta \rho_o$
as if the solid earth were rigid and the resultant elastic yielding
of the earth. The latter is related to $\Delta \rho_o$ through the load
Love numbers. Assuming a spherical earth, equation (7) be-
comes

$$\Delta A_{lm}(t) = \frac{1 + k_l'}{2l + 1} \int \Delta \rho_o(\Omega, t) Y_{lm}^{*}(\Omega) d\Omega$$

where $k_l'$ is earth’s load Love number of degree $l$; in general,
the $k_{l'}$ are negative because elastic yielding counteracts the
effect of $\Delta \rho_o$.

Equation (8) gives explicitly the changes in Stokes coef-
ficients of any degree $l$ and order $m$ of the gravitational field $U$
due to $\Delta \rho_o(\Omega, t)$. We shall first consider the low-degree zonal
harmonics with $m = 0$ and $l = 2, 3, 4$; they are the most
prominent components in $U$ for the earth. Zonal harmonics
are often expressed in terms of the $J_l$ coefficients given by

$$J_l = -(2l + 1)^{1/2} C_{ll}$$

From equations (2), (4), and (8), we obtain

$$\Delta J_{l}(t) = -\frac{1 + k_{l'}}{M} \int \Delta \rho_o(\Omega, t) P_l(\cos \theta) d\Omega$$

where $P_l$ is the Legendre polynomial of degree $l$: $P_l(\cos \theta) = (3 \cos^2 \theta - 1)/2$, $P_2(\cos \theta) = (5 \cos^2 \theta - 3 \cos \theta)/2$, and $P_3(\cos \theta) = (35 \cos^4 \theta - 30 \cos^2 \theta + 3)/8$. In this paper we shall adopt the $k_{l'}$ values from Farrell [1972]: $k_2' = -0.31$, $k_3' = -0.20$, $k_4' = -0.13$. Note that $P_l$ serves as a weighting
function in the integral: $\Delta \rho_o$ is weighted favorably around the
antinodes of the weighting function and unfavorably around
the nodes.

Changes in the earth’s rotation due to surface mass redistri-
bution can be readily obtained from equation (8) as well. Thus the excitation function \( \Psi \) of the polar motion is directly related to the complex Stokes coefficient of degree 2 and order 1 (for details see, e.g., Chao and Gross [1987]):

\[
\Psi(t) = \varPsi_x(t) + i\varPsi_y(t) = -\frac{\kappa}{2} \Delta A_{21}(t)
\]

where \( \varPsi_x \) and \( \varPsi_y \) denote the x and y components of \( \Psi \), respectively, and \( J_2 = 1.083 \times 10^{-3} \) is the earth's dynamic oblateness. Substituting into equation (10) the "transfer function" \( \kappa = 1/(1 + k^2) \) [Munk and MacDonald, 1960] and equation (8) with \( \gamma_{21} = 15^{1/2} \sin \theta \cos \theta \exp (i\lambda) \), we can now write equation (10) explicitly as

\[
\Psi(t) = -\frac{R^2}{M J_2} \int \Delta \sigma_\Omega(t) \sin \theta \cos \theta \exp (i\lambda) d\Omega
\]

Note that the weighting function in this integration has its antinodes at 45° latitudes and nodes at the equator and the poles. For convenience, we shall express \( \Psi \) in units of milliarc seconds (mas): 1 mas = 0.485 \times 10^{-8} \text{ radians}.

Anticipating its usage, we make the following qualitative observation derivable from equation (11): As a result of the geographical distribution of Eurasia and North America along opposite meridians, what excites the earth's polar motion is essentially the difference between the individual snow load contributions from these two continents. In other words, as far as \( \Psi \) is concerned, the two continental snow loads act to cancel each other, such that the continent with the greater snow load "pushes" the excitation pole toward the other hemisphere. Furthermore, because of the location of the continents along the 90°E and 90°W meridians, we should expect \( \Psi \), to be the dominant component of \( \Psi_x \) and \( \Psi_y \), to be negative at the time of northern winter if Eurasia has more snow than North America.

The change in length of day, \( \Delta \text{LOD} \), can be obtained from \( \Delta J_2 \) as follows. From the conservation of the z component of angular momentum, \( \Delta \text{LOD} \) is given by

\[
\Delta \text{LOD}(t) = \text{LOD} \Delta C(t)/C
\]

where \( C \) is the principal moment of inertia for the entire earth about the z axis. Here we are assuming that the core participates in the motion as a result of the core-mantle coupling on the seasonal time scale. This also facilitates comparison with previous studies. The same assumption has been made implicitly in equation (10) for the polar motion excitation.

By definition, the change \( \Delta C \) can be evaluated by

\[
\Delta C = (\Delta T + 2MR^2 \Delta J_2)/3
\]

where \( T \) is the trace of the earth's inertia tensor: \( T = 2\rho(r)^2 dV \) (for details, see Chao and Gross [1987] see G-G). Under the conservation of mass and the same condition that resulted in equation (8) (that the mass redistribution occurs only at the surface \( r = R \)), it is clear that \( T \) is a constant, i.e., \( \Delta T = 0 \). Therefore, combining equations (12) and (13), we have

\[
\Delta \text{LOD}(t) = \text{LOD} [2MR^2 \Delta J_2(t)]/(3C)
\]

or, in units of microseconds (\( \mu \text{s} \)), \( \Delta \text{LOD}(t) = 17.4 \times 10^{10} \Delta J_2(t) \). Note that expressions (11) and (14) are equivalent to equations (9.5.1a) and (9.5.1b) of Munk and MacDonald [1960], respectively.

The task, then, is to evaluate the integral in equation (8). For the snow load problem we shall make the assumption that in order to conserve water mass, the water stored as continental snow originates in the ocean (but see section 5.4). We further assume for simplicity that this results in a uniform decrease in the sea level over the entire ocean. Physically, the latter assumption requires a readjustment of ocean water to hydrostatic equilibrium on a time scale less than a month.

The surface integral in equation (8) can thus be conveniently partitioned into two parts: continental and ocean. Over the ocean, under the assumptions made above, \( \Delta \sigma_o \) is given by the negative of the total snow mass divided by the total ocean area. The resultant sea level decrease is then \( h_o(t) = -\Delta \sigma_o(t)/\rho_o \) where \( \rho_o = 1 \text{ g cm}^{-3} \) is the density of the water being removed from the ocean: a snow mass of \( 3.6 \times 10^{18} \text{ g} \) corresponds to an \( h_o \) of 1 cm. It is evident and has been shown in detail by B. F. Chao and W. P. O'Connor (1987) that, owing to the orthogonality of spherical harmonic functions, the integration in (8) for a uniform decrease of sea level depends only on the spherical harmonic coefficients of degree l and order m of the "ocean function" (which assumes value 1 over the ocean and 0 over the land; see Munk and MacDonald [1960]). Thus the contribution of the ocean variation to equation (8) can be readily evaluated. Adopting Lambeck's [1980] ocean function values, we find after some algebra that for a sea level decrease of \( h_o(t) \) centimeters, the ocean contributions are

\[
\Delta J_2(t) = -15.7 \times 10^{-12} h_o(t)
\]

\[
\Delta J_3(t) = 11.6 \times 10^{-12} h_o(t)
\]

\[
\Delta J_4(t) = -5.9 \times 10^{-12} h_o(t)
\]

\[
\Psi(t) = -1.68 - 2.14i h_o(t)
\]

\[
\Delta \text{LOD}(t) = -2.75 h_o(t)
\]

On continents we have \( \Delta \sigma_c(\Omega, t) = \rho_h h_s(\Omega, t) \), where \( h_s \) is the observed snow depth and \( \rho_h \) is the density of the snow which in reality is a function of depth and time. Here, without sufficient knowledge about how it varies, we shall take \( \rho_h = 0.3 \text{ g cm}^{-3} \) as an average value. The contribution of the continental snow to equation (8) will be evaluated by a grid summation over the land area using data to be described in the next section.

3. DESCRIPTION OF DATA

The snow depth data set under study is derived from passive microwave remote-sensing observations made by the scanning multichannel microwave radiometer (SMMR) instrument onboard the Nimbus 7 satellite. It was launched on October 24, 1978, into a sun synchronous polar orbit (inclination 89°) with local noon and local midnight equatorial crossings. SMMR operates at five different microwave frequencies while scanning 25° to either side of the spacecraft with a constant incidence angle of approximately 50° with respect to the earth's surface. The spatial resolution varies from 25 to 150 km depending on the frequency (for more detail, see Gloersen and Barath [1977]).

The SMMR receives microwave emissions from all terrestrial sources including the snow volume and the underlying ground. Snow particles act as scattering centers for microwave radiation from a snowpack. The scattering redistributes the upwelling radiation according to snow thickness and crystal size, providing the physical basis for microwave detection of
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Fig. 1. Time series showing the variations caused by snow for 1979-1985: (a) total snow mass and corresponding sea level decrease, (b) \( \Delta J_2 \) and \( \Delta \text{ALOD} \), (c) \( \Delta J_3 \), (d) \( \Delta J_4 \), (e) \( \varphi_6(f) \) \( \psi_6 \). The mean values in the polar motion excitation functions \( \varphi_6 \) and \( \psi_6 \) have been removed.

snow [e.g., Foster et al., 1984]. Assuming spherical snow crystals with a mean grain size of 0.35 mm and a snow density of 0.3 g cm\(^{-3}\), a radiative transfer technique using Mie scattering theory has been used to derive a relation that gives the snow depth using the brightness temperatures of the snow at two SMMR frequencies, namely, 18 and 37 GHz [Chang, 1986]. The resulting snow depths are reliable up to a depth of about one meter.

In this paper, we have analyzed 7 years of data spanning the period 1979–1985. In its present form, the data set consists of monthly mean snow depths, each obtained by averaging data from approximately 200 night orbits. The grids are 0.5° in longitude and latitude, covering the northern hemisphere land area from the equator to 85°N. On Greenland the snow is underlain by glacial ice (as opposed to ground). The glacial ice extends much deeper than the microwave penetration depth, and our present technique is unable to resolve the newly fallen snow from the ice. The lack of SMMR data north of 85°N where no land exists does not concern us because (hydrostatically) floating ice has practically no effect on the earth's rotation and gravitational field.

No data have been reduced for the southern hemisphere at the present time. In the southern hemisphere the only extensive snow field is found on Antarctica where, like Greenland, the thick continental ice sheet would have precluded the usage of the snow depth determination technique. The lack of Antarctic and Greenland snow data does, to different extents, introduce inadequacies in our present study. Outside Antarctica the southern hemisphere has very little snow, and the exclusion of these snow fields should not be critical. This will be discussed further in sections 5.3 and 5.4.

4. RESULTS

Figure 1 exhibits our calculated time series for the period 1979–1985. The seasonal cycles are evident. To each time series we have performed a least squares fit using as base functions an annual sinusoid and its higher harmonics including semiannual and one-third year components. The numerical estimates quoted below are based on these least squares fits. Note that the inclusion of any one component for fitting does not interfere with the fit for any other components because of the orthogonality of the base functions in the studied time span which consists of an integer number of years (and hence cycles).

Figure 1a shows the total snow mass over the land area that we have data for, assuming a mean snow density of 0.3 g cm\(^{-3}\). In an average year, the snow accumulation culminates in February–March with a total mass of \( 3.3 \times 10^{18} \) g, and dwindles to less than \( 0.02 \times 10^{18} \) g in July–August. The corresponding decrease in the sea level is shown on the right-hand scale.

The changes in \( J_2, J_3, \) and \( J_4 \) are shown in Figures 1b, 1c, and 1d, respectively. It is seen that \( \Delta J_2 \) shows the greatest variation. The least squares fit gives \( \Delta J_2 \approx (-0.638 \sin \omega t - 0.983 \cos \omega t - 0.249 \sin 2 \omega t - 0.014 \cos 2 \omega t) \times 10^{-9} \), where \( \omega = 2\pi/(365 \text{ days}) \) and the nominal origin of time \( t = 0 \) is mid-January. The annual (peak-to-peak) amplitude is then \( 2.3 \times 10^{-9} \) and the semiannual amplitude is \( 0.50 \times 10^{-9} \). By comparison, the secular rate of change in \( J_2 \) determined from laser ranging of the LAGEOS satellite [Yoder et al., 1983; Rubincam, 1984] is about \( 3 \times 10^{-11} \) per year.

It is also noticed that the variation in \( \Delta J_3 \) decreases rapidly as \( l \) increases from 2 to 4. This trend is expected to continue for higher degrees in light of the unimodal distribution of the snow mass with respect to latitude. The estimated (peak-to-peak) annual amplitude for \( \Delta J_3 \) is \( 1.1 \times 10^{-10} \) and the semiannual amplitude is \( 0.18 \times 10^{-10} \). The variation in \( \Delta J_4 \) exhibits relatively strong semiannual amplitude: \( 0.23 \times 10^{-10} \) compared to its annual amplitude of \( 0.22 \times 10^{-10} \).

The change in length of day (ALOD), being proportional to \( \Delta J_2 \) according to equation (14), is shown on the right-hand scale of Figure 1b. The annual amplitude of ALOD is 41 \( \mu \)s.

Fig. 2. The Hann-windowed power spectrum of the snow load excitation function of polar motion, 1979–1985. The decibel scale is relative to 1 mas\(^2\)/cpy.
This is below the detection threshold of modern geodetic techniques.

Figures 1e and 1f show the x and y components, respectively, of the polar motion excitation function \( \Psi(t) \). The mean value in \( \Psi(t) \), being geophysically uninteresting, has been removed. Note that the polarity of \( \Psi_- \) and the fact that \( \Psi_+ \) exhibits the greater variation are as expected (see section 2). Figure 2 displays the Fourier power spectrum (or periodogram) of the complex \( \Psi(t) \). We have pre-multiplied \( \Psi(t) \) by a Hann (or cos^2) window before the Fourier transformation in order to reduce spectral leakage. The positive frequency represents \( \Psi_+ \), the component of \( \Psi \) that circles about the north pole in the same sense as the rotation of the earth (a property hereinafter called prograde); and the negative frequency represents \( \Psi_- \), the component of \( \Psi \) that circles in the opposite sense (hereinafter called retrograde) [see Munk and MacDonald, 1960, p. 47]. Figure 2 clearly shows the dominant annual signal in \( \Psi(t) \) as well as many higher harmonics.

Among these seasonal signals, we shall only concentrate on the annual wobble (particularly its prograde component \( \Psi_+ \)) because the higher harmonics, being distant in frequency from the earth's natural Chandler frequency, are dynamically suppressed and have rather low signal-to-noise ratio in polar motion observations. To extract the annual signal from \( \Psi(t) \), our least squares fit yields \( \Psi_+(t) = 1.350 \sin 0t - 2.923 \cos 0t \) (mas), and \( \Psi_-(t) = -3.626 \sin 0t - 8.385 \cos 0t \) (mas). We then algebraically convert \( \Psi_+ \) and \( \Psi_- \) into the prograde component \( \Psi_+ \) and the retrograde component \( \Psi_- \) [Munk and MacDonald, 1960, p. 47]. To facilitate comparison with previous estimates, \( \Psi_+ \) and \( \Psi_- \) are then rotated "backward" in the complex plane by 15° (clockwise for \( \Psi_+ \) and counterclockwise for \( \Psi_- \)) to bring the origin time \( t = 0 \) to January 1. The final results are

\[
\begin{align*}
\Psi_+ &= -1.6 - 4.6i \text{ mas} \quad \text{or} \quad (4.9 \text{ mas}, -109°) \quad (16a) \\
\Psi_- &= 4.2 - 2.3i \text{ mas} \quad \text{or} \quad (4.8 \text{ mas}, -28°) \quad (16b)
\end{align*}
\]

The 5 years of data (1980-1984) analyzed by Chao et al. [1987] yielded annual wobble excitation results only slightly different from the above, indicating that the snow load distribution is relatively consistent in its annual cycle during the period studied.

The prograde and retrograde vectors \( \Psi_+ \) and \( \Psi_- \) given by equation (16) are labeled SNOW and plotted in Figures 3a and 3b, respectively. Their almost equal amplitudes are a consequence of the fact that the annual \( \Psi_+ \) and \( \Psi_- \) stay virtually in phase. Thus the snow load annual wobble excitation pole does not have much of a rotational motion.

5. DISCUSSION

5.1. Comparison of Annual Wobble Excitations (Figure 3)

Annual wobble excitation requires a more detailed discussion with regard to previous investigations and existing observations. In Figure 3 we have also plotted other annual \( \Psi^z \) estimates for references. The vectors labeled "ILS" indicate those inferred by Wilson and Vicente [1980] from the International Latitude Service observations for the period 1900-1977. "LAGEOS" indicates those from the LAGEOS satellite laser ranging observations for the period 1977-1985: \( \Psi_+ = 6.56 - 14.75i \) mas, \( \Psi_- = -4.21 - 6.20i \) mas (determined by R. S. Gross [personal communication, 1986]). Note that "ILS" and "LAGEOS" agree quite well on their \( \Psi_+ \) estimates but not on their \( \Psi_- \) estimates. This is of course because

\( \Psi_- \) is much less accurately established owing to the lower signal-to-noise ratio in the retrograde annual wobble. Consequently, less emphasis should be put on \( \Psi_- \) when polar motion observations are involved.

The vectors "ILS" and "LAGEOS" are the astrometric observations of annual wobble excitation for which geophysicists are seeking complete explanations. Presumably (see section 1), a major part has its origin in the atmosphere; but it is evident from Figure 3 that the snow load contribution is indeed considerable: for \( \Psi_+ \) it has a magnitude of about 30% of the observed value while maintaining a phase lag of about 60° behind the observed.

The vectors labeled VH in Figure 3 are the annual \( \Psi^z \) obtained by Van Hylckama [1970] based on his revised data set for continental water storage (of which the snow is a component). An earlier version of Van Hylckama's [1956] data set, used by Munk and MacDonald [1960] and only of historic interests now, yields estimates that are not much different from VH. These are the only published estimates of their kind that are widely quoted. In recent calculations, however, Hinnov and Wilson [1987] puts their annual \( \Psi^+ \) estimate in the same quadrant as VH but with a considerably larger magnitude, while Close and Stolz [1987] obtain annual \( \Psi^z \) estimates that are some 90° clockwise from VH.

It is interesting that our annual \( \Psi^+ \) vector for snow points to almost the same direction as those determined by Wilson and Haubrich [1976] and Wahr [1983] for the air mass. This can be explained by the fact that both the air mass and snow load have similar seasonal and geographical distributions, with maximums over Asia in winter. Note also that in both prograde and retrograde cases our SNOW vector is in almost the opposite direction to VH. We shall return to this in the next section.

5.2. Comparison With Rand Corporation Data

As an attempt to acquire an independent check on the results of the above satellite data, we have also analyzed a global snow depth data set gathered by conventional climatological means: the data set published by the Rand Corporation and distributed by the World Data Center A for Glaciology [Schutz and Bregman, 1979]. It consists of 12 mean-
monthly snow depth values for each 4° of latitude by 5° of longitude grid square, obtained from a number of climatological references, with extrapolations for data sparse regions. No detailed description of their methodology is available.

First, we examined the snow mass (again assuming $p_s = 0.3 \text{ g cm}^{-3}$) on the northern hemisphere continents of Eurasia and North America (divided conveniently by the Greenwich Meridian/International Dateline). It was found that according to the Rand data, the total amount of snow in any given month is almost evenly divided between Eurasia and North America and only during December and January does Eurasia account for more than half of the total snow. Although this is unrealistic, we proceeded nevertheless.

Compared with the snow data from SMMR, the Rand data set was found to give about twice as much snow in the annual cycle. As a result, the same is true for the variations in the $J_l$ coefficients and the length of day. The situation with polar motion excitation $\Psi(t)$ is more complex: Contrary to the SMMR estimates (Figures 1e and 1f), the Rand result does not exhibit any seasonal behavior. In fact, the Rand data dictate that only in February and October does Eurasia have greater snow load contribution to $\Psi$, than North America (resulting in a negative total $\Psi$) and that in the month of January there is a more than 99% cancellation between the contributions to $\Psi$ from the two continents. These problems were found to be in part due to the great amount of snow fall the Rand data set bestows on Greenland, nearly 30% of the global total. So we removed Greenland from the Rand data set, and a more realistic, seasonal behavior for $\Psi$ emerged.

However, we recall the competition between the North American and Eurasian snow loads to determine the net $\Psi$ (see the discussion following equation (11)). The result is the difference of two large numbers, and a small error in the snow estimation can be greatly magnified in $\Psi$. We believe this is indeed happening with the Rand data and decided not to look into Rand results any further.

The moral is that the importance of accurate data in estimating $\Psi$ cannot be overemphasized, and this is where satellite data can be of great value. A corollary is that the validity of Van Hylckama's estimates, VH in Figure 3, should really be reviewed critically, as indeed proposed by Close and Stolz [1987]. Thus it remains to be determined whether the snow contribution to the annual wobble excitation (SNOW in Figure 3) is truly opposite to (or, for that matter, in any other relation with) the total water storage contribution. At any rate, we believe that our determination of the SNOW vectors represents one step closer to the solution of the annual wobble excitation problem.

5.3. Antarctica and Greenland Problem

As said in section 3, the continental ice sheets on Antarctica and Greenland cause problems in our present study because their presence beneath the seasonal snow precludes the usage of our snow depth determination technique. A more complex problem, however, arises as the ice sheets themselves participate in the redistribution of continental ice/snow mass in the form of glacial movements: the ice sheet acts as a "conveyor belt" that carries seaward the snow accumulation. For instance, in Antarctica, which has an area of about $13 \times 10^6 \text{ km}^2$, the snow accumulation amounts to an average of about 15 cm of water equivalent per year with very little summer melting [NASA, 1985]. Because of this "conveyor belt" effect, the apparent snow depth (accumulated over the years) no longer represents the actual net mass distribution. Unfortunately, the nature of the Antarctic ice/snow mass balance is virtually unknown. We do not know how close it is to a steady state on a time scale longer than seasonal periods; and more important to the present study, we do not know the amount of seasonal modulation on the mass balance. New types of data (for example, a long-term altimetric monitoring with centimeter accuracy) will be needed to shed light on this problem [NASA, 1985].

At present then, all we can hope for is an order-of-magnitude estimate. Let us do the following exercise. Suppose Antarctica has an annual snow cycle which varies with a peak-to-peak amplitude of 1 cm of water equivalent and is uniform over the entire continent. It should be pointed out here that the 1-cm amplitude is probably an overestimate because any such seasonal modulation must owe its origin to evaporation, as any surface meltwater generated during warm periods would be likely to percolate into the snow and re-freeze. Suppose further that Antarctica is a circular land mass covering all the area south of 70°S latitude. It is clear from equation (8) that such an axially symmetric arrangement only has nonzero contributions to the zonal coefficients $J_l$ in particular, its contribution to $A_{21}$, and hence polar motion excitation $\Psi$, is zero identically.

As before, using equation (8) and allowing for the oceans (which according to equation (15) turn out to have relatively minute contributions in the present case), we find $\Delta J_2 = -0.17 \times 10^{-10}$, $\Delta J_3 = 0.18 \times 10^{-10}$, and $\Delta J_4 = -0.17 \times 10^{-10}$. The change in LOD follows immediately from $\Delta J_2$ (equation (14)): $\Delta LOD = 3.0 \mu s$. Notice that unlike the northern hemisphere and because the south pole is an antinode for all zonal harmonics, the above Antarctic contribution to low-degree $\Delta J_l$ does not decrease with respect to the degree $l$. Therefore it plays a progressively more important role as $l$ increases; in our exercise it is less than 7% of the northern hemisphere contribution for $\Delta J_2$ but amounts to almost 40% for $\Delta J_4$. Note also that the Antarctic contribution is in phase with and augments in absolute values to that from the northern hemisphere if $l$ is even, while the reverse is true if $l$ is odd.

In reality, of course, Antarctica and its snow are not axially symmetric. This invalidates the statement that the Antarctic contribution to the polar motion excitation $\Psi$ is zero. However, the Antarctic $\Psi$ should still be small indeed because its polar location would hinder the excitation of polar motion (cf. equation (11)). On the other hand, the departure from axial symmetry only introduces small errors in the computation of low-degree $J_l$; it is the amount of seasonal snow variation that is the major uncertain factor.

The situation with Greenland is quite different. Greenland is only about one sixth as large as Antarctica. It lies $70^\circ$ off the north pole and spans some 40° in longitude. The average snow accumulation rate is about 30 cm of water equivalent per year, and there is substantial summer melting in coastal regions [NASA, 1985]. However, the ice/snow mass balance and its seasonal modulation are also largely unknown.

The Greenland contribution to the snow load changes in $J_l$ coefficients should be much smaller than Antarctica's. Its importance will not grow as $l$ increases because of Greenland's off-pole location and its north-south elongated shape. On the other hand, its contribution to the polar motion excitation $\Psi$ is harder to assess without actual observations. Qualitatively, it would detract from $\Psi$, decreasing its amplitude in light of the Eurasia versus North America competition for $\Psi$ (see sec-
tion 2). However, considering Greenland's small area and its high latitude which would make the excitation of polar motion difficult, we claim that the exclusion of Greenland is tolerable in calculating $\Psi$.

5.4 Southern Hemisphere Problem

The present lack of snow data for the southern hemisphere (outside Antarctica) is by comparison a much more benign problem simply because of the small amount of continental snow involved. The only substantial snow field is in South America, where the area covered by seasonal snow appears to be quite variable from year to year. But even in the snowy year of 1980, the snow cover only reached a maximum in excess of $1 \times 10^8$ km$^2$ [Dewey and Heim, 1983]. This is only about 1/40th of the snow cover of the northern hemisphere and hence contributes little to the overall calculation. Other snow fields found in southeast Australia, New Zealand, and the southern tip of Africa are simply too small (and often too infrequent) to warrant any serious consideration [Dewey and Heim, 1983].

It is nevertheless worthwhile to examine in some greater detail the contribution of South American snow to the excitation of polar motion. Situated in the western hemisphere but south of the equator and 180° out of phase in season, South American snow "joins force" with North American snow in counteracting the dominating snow in Eurasia in the polar motion excitation. Thus, as far as $\Psi$ is concerned, our negligence of the South American snow contribution has the same kind of effect, with probably comparable amplitude, as the contribution from Greenland snow.

5.5 Role of Atmospheric Water

In section 2 we have assumed for simplicity that the continental snow accumulates at the expense of ocean water, and the loss of ocean water contributes to our calculations according to equation (15). It was found that this ocean contribution is in general only a small fraction of the contribution from the continental snow, for example, ~5% for $\Delta J_2$ (and hence $\Delta LOD$), ~10% for $\Psi_r$, and ~15-30% for $\Psi_c$.

It can be argued that part of the snow mass comes from sources other than the ocean. After all, the assumed exchange of water between the ocean and the continental snow is accomplished by way of atmospheric transport in the form of water vapor (and to a lesser extent, clouds). In fact, it has been known that the total atmospheric water content reaches a minimum in northern winter, which coincides with the maximum in snow mass. Trenberth [1982] has estimated the total amount of the atmospheric water variation to be about $1 \times 10^{18}$ g, or less than one third of the maximum snow mass. For this reason, we may neglect any explicit treatment of the atmospheric water vapor contribution which should amount to no more than a few percent. A stronger reason, nevertheless, is that the atmospheric water vapor is actually included in the air mass calculation [e.g., Wilson and Haabrich, 1976] because water vapor contributes to the reading on a barometer. If all investigations of individual water components (e.g., atmospheric water, soil moisture) simply refer to, say, the ocean as the ultimate water source and insist on the conservation of water mass (as we have done in this paper for snow), a consistent treatment can be achieved.

5.6 Chandler Wobble Excitation

The dominant share of the spectral power in the polar motion excitation $\Psi$ by the snow load resides at seasonal frequencies (Figure 2). Now let us study the part of the spectral power in $\Psi$ that is responsible for exciting the Chandler wobble.

The Chandler wobble is the earth's version of the Eulerian "free precession." Its major source of excitation has not been identified as yet. The Chandler wobble has a period of about 435 days, or a frequency of 0.84 cycles per year (cpy). The snow load contributes to the Chandler excitation through interannual variations in both its timing and its geographical distribution. In order to reveal the power spectrum at the Chandler frequency, it is necessary to remove the dominant seasonal, especially the annual, signals. The beat period between the annual and the Chandler wobbles is about 6.2 years. This is the minimum time span of the data required to resolve the two wobble frequencies spectrally. Our 7-year data span meets the requirement.

Removal of seasonal signals is done by subtracting from $\Psi(t)$ the least squares fit of a combination of annual, semiannual, and one-third year sinusoids. The resultant time series is then Hann windowed and its power spectrum calculated. Shown in Figure 4 (curve b) between -1.5 and 1.5 cpy, this is the power spectrum of the Chandler excitation due to snow load. Compared with Figure 2, the power spectrum value at the Chandler frequency is about 26 dB less than the annual power.

Figure 4 (curve a) shows the power spectrum, also Hann windowed, of the observed Chandler excitation. The latter is derived from a Bureau International de l'Heure (BIH) polar motion data set for the same period, 1979-1985, adopted from B. F. Chao (A correlation study of the earth's rotation with El Nino/Southern Oscillation, submitted to Journal of Geophysical Research, 1987). It is seen that at the Chandler frequency the power of the snow load contribution to the Chandler excitation is about 25 dB less than the power needed to maintain the observed Chandler wobble during the studied period. However, at present it is not clear how realistic this estimate is because of the admittedly low signal level of the snow variation at the Chandler frequency. Furthermore, the contribution to the Chandler excitation from the southern hemisphere snow, which is lacking in our data set, is presently unknown. It may not be substantially smaller than its northern hemisphere counterpart simply because of its noted interannual variability (see section 5.4). In order to yield Chandler excitation estimates that are more reliable, a much
longer and improved data set with higher temporal resolution is necessary.

5.7. Future Plans

In the near term we intend to improve the calibration accuracy of our Nimbus 7 SMMR snow depth observations, as well as introduce a more realistic snow density model (giving \( \rho_s \) as a function of depth, for instance). We will also deduce data for the southern hemisphere and improve the temporal resolution of the data from a month to perhaps a few days. Continuation of the microwave data is expected from the Defense Meteorological Satellite Program, Special Sensor Microwave Instrument (DMSP-SSM/I) due to be launched in 1987 (e.g., J. L. Foster et al., Global snow cover and the earth's rotation, paper to be presented at IUGG General Assembly, Vancouver, British Columbia, Canada, August 1987). While SMMR on Nimbus 7 continues to function, any overlap of these two data sets can be used to improve further the calibration. In the long term, we intend to look into other satellite remote-sensing data sets that bear importance to the water storage excitation of the polar motion. For example, global rainfall, atmospheric water vapor, soil moisture, and the seasonal biomass variation (which also affects the amount of evapotranspiration) may possibly be deduced from existing satellite observations. Finally, we reiterate that a long-term altimetric monitoring of ice sheets may prove valuable in observing the water balance on Antarctica and Greenland.

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